MAGIC SQUARES

I and II

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1. Introductory Note

By dedicating the 6th and 7th issues of the Science and Technology series to Magic Squares, Macao Post intends not only to promote this theme in scientific and cultural terms, but to create also a unique product in the history of Philately, i.e. a Magic Square formed by stamps specifically produced for this purpose. The issue Magic Squares I was published on 9th October 2014, World Post Day, whereas the issue Magic Squares II was published on 12th of November 2015.

In the Magic Squares I issue, besides the Souvenir Sheet based on the Luo Shu Magic Square, it was issued a Sheetlet with the Shape of a Matrix with 3 Rows \times 3 Columns (9 stamps), a Luo Shu Magic Square replica, composed of six stamps: three occupying the Superior Row, with the face values of 4, 9 and 2 “patacas” and the remaining three, occupying the Central Row, with the face values of 3, 5 and 7 “patacas”. The Inferior Row was left intentionally empty to accommodate the remaining three stamps to be published in the Magic Squares II issue.

In the Magic Squares II issue, it was also issued a Sheetlet with the Shape of a Matrix with 3 Rows \times 3 Columns (9 stamps), a Luo Shu Magic Square replica, composed of three stamps with the face values of 8, 1 and 6 “patacas” occupying the Inferior Row. The Superior and Central Rows were left intentionally empty to accommodate the six stamps published in the Magic Squares I issue.

Therefore, it is possible to build, taking as basis the first issue Sheetlet (adding to it the last three stamps issued) or the second issue Sheetlet (adding to it the first six stamps issued), a Luo Shu Magic Square made of stamps.

The interest for Magic Squares is transversal in Chinese and Western Cultures. The importance of this theme in Chinese culture can be traced back to ancient times of its history which is shown by the myths illustrated in the Luo Shu Souvenir Sheet of Magic Squares I issue, as well as the Magic Circles created by Yang Hui presented in the First Day Cover of Magic Squares II issue.

2. Brief Early History

According to some researchers and academics the early history of Magic Squares is not well researched neither documented. However, there is a general view that three civilizations have contributed to its creation: the Chinese, the Indian and the Arab.

The first recorded Magic Square, an Order 3, is known as Luo Shu or Luo River Scroll, was presented in detail in the Souvenir Sheet of Magic Squares I issue. Later, Magic Squares and Magic
Circles appeared in Chinese Literature around the Northern Song Dynasty (960-1127).

Some researchers thought that Magic Squares travelled from China to India, where several Magic Squares of Order 4 were presented by the Buddhist Philosopher Nagarjuna, who lived about the Second Century, in his work Kaksaputa. It took more than 1000 years to find a Jaina Inscription that showed, again, an Order 4 Magic Square, although with special properties not yet present in the works of his predecessor. This kind of Magic Squares was named lately Jaina Magic Squares.

However, it is due to the Mathematician Narayana Pandit a systematic study of Magic Squares that, in his work Ganita Kaumudi (1356), presented general methods for the construction of several sorts.

From India, probably, Magic Squares travelled to Arabic world and it is known that Islamic and Arabic Mathematicians took awareness of Magic Squares before the Seventh Century. So, it was not surprise that Magic Squares of Order 5 and 6 appear in Rasa’il Ikhwan al-Safa, Encyclopedia of the Brethren of Purity, composed by 52 Epistles in Mathematics, Natural Sciences, Psychology and Theology, in Baghdad, around the year 983. Later, around 1200, Ahmad al-Buni, a famous Arab Mathematician, also studied Magic Squares.

Arabs do not only use Magic Squares in making astrological calculations and predictions but also, like Ahmad al-Buni, believe in their mystical properties.

The introduction of Magic Squares in Europe is believed to take place, not only, in 1280, through a Spanish Manuscript where to each one Magic Square a Planet was associated, as in the Islamic Literature, as well as, around year 1300, when the Greek Byzantine, Manuel Moschopoulos (1265-1316) wrote a mathematical treatise based on the works of Ahmad al-Buni. Magic Squares of Order 6 and 9 appeared again in the XIV Century, in Florence, in a manuscript of the (Trattato d’Abbaco) Treatise of the Abacus by Paolo Dagomari, Mathematician, Astronomer and Astrologer. Following Moschopoulos, Luca Pacioli (1445-1517), fellow Florentine like Dagomari, also studied Magic Squares and presented in his work De Viribus Quantitatis, around 1500, several of Order 3 to 9.

Following these pioneers, during XV to XVII Centuries, the most notable names in working or developing Methods for solving Magic Squares were, namely: Albrecht Dürer (1471-1528), whose engraving Melancholia I, an Order 4 Magic Square, was also introduced in Magic Squares I issue ; the German Physician and Theologian Heinrich Cornelius Agrippa (1486-1535), who wrote in 1510 “De Occulta Philosophia, Libri Tres”, at that time one of the most influential books around Europe in which he exposed, with examples, the mystical powers of Magic Squares of Orders 3 to 9; Johann
Faulhaber (1580-1635); Claude Gaspar Bachet de Meziriac (1581-1638); Simon de la Loubere (1642-1729) whose Method of Construction of Magic Squares was introduced in a stamp in Magic Squares I issue.

Over the years several mathematicians as Pierre de Fermat (1601-1665), Bernard Frenicle de Bessy (1605-1675), Benjamin Franklin (1706-1790) presented in the stamp Bent Diagonals in the Magic Squares I issue, Leonhard Euler (1707-1783), Arthur Cayley (1821-1895) etc. have shown interest to study Magic Squares discovering interesting relations and properties.

3. Magic Squares: Basic Definitions

A – Numeric Magic Squares

3.01 Cell – Each one of \( n^2 = n \times n \) small squares that constitute the \( n \) Rows and \( n \) Columns of the square. Cells can be filled with Numbers, Letters/Characters or Geometric Shapes/Pieces.

3.02 Closed Magic Line – Lines that connect the centers of Cells in numeric sequence, including the return to the first, e.g. 1, 2, 3, \( \ldots \), \( n^2 \ldots 3 \), 2, 1 for a Normal Magic Square (after Claude Bragdon). When the areas between the Magic Line are filled with different colours, some very interesting patterns may be created, called Sequence Patterns (after Jim Moran).

3.03 Column – Set of \( n \) vertical Cells. A square of Order \( n \) has \( n \) Columns.

3.04 Complementary Pair or Complementary Numbers – Pair of Cells or Pair of Numbers which sum is equal to the sum of the first and last terms of the series, e.g. \( 1+n^2 \) for a Normal Magic Square.

3.05 Diagonals

3.05.1 Bent Diagonal – Diagonal, with the same \( n \) number of Cells as the Order \( n \) of the Square, that:
- Starts at any edge Cell of the Square and finish at the opposite edge Cell;
- Is composed by two V-shape perpendicular and symmetric segments with the vertice coinciding with: the center of the Square, for Even Order; the center of the central Cell, for Odd Order.

There are \( 4 \times n \) Bent Diagonals, distributed as follows:
- Even Order: Continuous, \( 2(n+2) \), Wrap-Around, \( 2(n-2) \);
• **Odd Order**: Continuous, $2(n+1)$, Wrap-Around, $2(n-1)$.

  Bent Diagonals also sum the Magic Sum $S$ and are the main characteristic of Benjamin Franklin “Magic Squares”.

3.05.2 **Broken Diagonal Pair** – Two Short Diagonals – with the same or different number of Cells – that are parallel to a Main Diagonal, but on opposite sides, and when connected, contain the same number of Cells, as each Row, Column or Main Diagonal, i.e. the Order $n$ of the Magic Square.

  Broken Diagonals also sum the Magic Sum $S$ and are the main characteristic of the Pandiagonal Magic Squares.

3.05.3 **Main Diagonal** – Each one of the two diagonals, called Leading or Left Diagonal and Right Diagonal, constituted with $n$ Cells, which connect the opposite corners of the square.

3.05.4 **Opposite Short Diagonals Pair** – Two Short Diagonals which are parallel and on opposite sides of a Main Diagonal and contain the same number of Cells.

3.05.5 **Pandiagonal** – That include all diagonals: Main Diagonals and Broken Diagonals.

3.05.6 **Short Diagonal** – Diagonal that is parallel to one of the Main Diagonals and intercept two adjacent sides of the square. A Short Diagonal may have 1 to $n-1$ Cells.

3.05.7 **Wrap-Around Diagonal** – Bent Diagonal obtained through Wrap-Around process.

3.06 **Magic Sum** or **Magic Number** or **Magic Constant** – The constant sum of the $n$ Cells of each Row, Column, Main Diagonal, etc. For a Normal Magic Square the Magic Sum $S$ can be calculated by the formula $S = n(n^2+1)/2$, where $n$ is the Order of the Magic Square.

3.07 **Normal** or **Pure** or **Traditional Magic Square of Order n** – Magic Square where the numbers used for filling the $n^2$ Cells are consecutive positive integers from 1 to $n^2$.

3.08 **Order n** – There are four classes of Magic Squares according the Order:

  • **Even** – When $n$ is even, i.e. $n = 2, 4, 6, 8…$;
  • **Odd** – When $n$ is odd, i.e. $n = 3, 5, 7…$;
  • **Doubly–Even** – When $n$ is multiple of 4, i.e. $n = 4, 8, 12…$;
  • **Singly or Oddly–Even** – When $n$ is even but is not divisible by 4, i.e. $n = 6, 10, 14…$
3.09 **Row** – Set of $n$ horizontal Cells. A *Square of Order* $n$ has $n$ Rows.

3.10 **Square of Order** $n$ – Square composed with $n$ Rows, $n$ Columns and $n^2$ Cells.

3.11 **Symmetrical Pair** or **Symmetrical Cells** – Pair of Cells diametrically equidistant or symmetric in relation to the Center of the Square.

3.12 **Wrap-Around** – To connect the square opposite sides (i.e. left-right or up-down) in order to make a continuous cylindrical surface where the opposite sides overlay.

B – Letters/Words/Characters Magic Squares

3.13 **Palindrome** – A number, word or phrase that can be read the same in different directions (e.g. right-to-left or left-to-right). However, as it is the case for the *Su Hui Xuan Ji Tu Palindrome* poem, the verses, in Chinese language, can also be read in different directions, not with the same meaning, but yet meaningful.

C – Geometric Magic Squares

3.14 **Geometric Magic Square** or **Geomagic Square** – An array of $n^2$ Cells ($n$ Rows $\times n$ Columns) each occupied by a distinct geometrical (usually planar) Shape or Piece, such that $n$ of them taken from any Row, Column or Main Diagonal can be assembled to create a Larger Constant Shape or Piece known as the Magic Target. By the **Dimension** of a Geomagic Square is meant the dimension of its Shapes or Pieces.

3.15 **Magic Target** – The Larger Constant Shape or Piece formed by the union of the $n$ Shapes or Pieces occurring in any Row, Column, or Main Diagonal. The Shapes or Pieces used are usually planar, but may be of any dimension. 3-D Pieces will thus assemble to form 3-D Targets, while 1-D Shapes are simply straight line segments (each of which could alternatively be represented by a single number). The Magic Target will then be another straight line (equal in length to the sum of those numbers). From this is seen that Numerical Magic Squares are in fact a particular kind of Geomagic Squares.

4. **Magic Squares: General Classification**

The *Magic Squares* can be categorized, according to special properties they may have, in
different ways. When the Cells of the square are filled with Numbers, Letters/Characters or Geometric Shapes, the square is called, respectively, Numeric Square, Letters/Words/Characters Square and Geomagic Square (after Lee Sallows).

One of the most frequent classifications divides them in three classes: Simple, Associated and Pandiagonal. However, removing or adding them some properties, we can also consider in addition the following three: Semi-Magic, Semi-Pandiagonal and Most Perfect.

When they show special characteristics, take others names as: Bimagic, Bordered or Concentric, Inlaid, Alfamagic, Latino, Domino, IXOHXI, Prime, Serrated, etc.

The previous six classes are defined, for Magic Squares of Order n, in the following paragraphs 4.1 to 4.6, according the increasing complexity of their properties.

4.1 **Semi-Magic Square** – Array of \( n^2 \) Cells \((n \ \text{Rows} \times \ n \ \text{Columns})\) where the sum of each Row and of each Column is equal to the Magic Sum \( S \). The sum of one or the two Main Diagonals is different of \( S \) and hence the name Semi-Magic.

4.2 **Simple, Normal, Numeric Magic Square of Order n** or simply **Magic Square** – Array of \( n^2 \) Cells \((n \ \text{Rows} \times \ n \ \text{Columns})\) each one filled with a certain number in a way that the sum of each Row, each Column and the two Main Diagonals all have the same value \( S \), called Magic Sum.

The fulfillment of the above mentioned properties is the minimum requirement to qualify as a Magic Square.

4.3 **Associated** or **Regular** or **Symmetrical Magic Square** – Magic Square where the sum of all Symmetric Pairs is equal to the sum of the first and last terms of the series, i.e. \( 1 + n^2 \).

In an **Odd Order Associated Magic Square**, the Center Cell is always equal to the middle number of the series, i.e. \((1+n^2)/2\).

No **Singly-Even Order Associated Magic Square** exists.

4.4 **Semi-Pandiagonal** or **Semi-Diobolic** or **Semi-Nasik Magic Square** – Magic Square with the following properties:

**Even Order:**
- The sum of an Opposite Short Diagonal Pair with \( n \) Cells, is equal to the Magic Sum, \( S \).

**Odd Order:**
- The sum an Opposite Short Diagonal Pair with \( n-1 \) Cells plus the Center Cell, is equal to the Magic Sum, \( S \);
- The sum of an Opposite Short Diagonal Pair with \( n+1 \) Cells minus the Center Cell, is equal to the Magic Sum, \( S \).

For a Magic Square of Order \( n \), the Magic Sum \( S \), is always calculated taken into account the \( n \) Cells of the **Rows, Columns and Pandiagonal Diagonals**. This is the reason why it is necessary to add or subtract the Center Square Cell to the Opposite Short Diagonal Pair Cells, when these are, respectively, \( n-1 \) and \( n+1 \).
4.5 Pandiagonal or Diabolic or Nasik or Continuous Magic Square - A Magic Square where each Broken Diagonal Pair sum is equal to the Magic Sum, S. The Pandiagonal Magic Square is considered one of the most sophisticated among the classes of Magic Squares; No Singly-Even Order Pandiagonal Magic Square exists.

4.6 Most Perfect Magic Square - Pandiagonal Magic Square of Doubly-Even Order, with the two additional properties:

• The Cells of any square of Order 2, (2 × 2 Cells) extracted from it, including Wrap-Around, sum the same value, \(2(1+n^2)\);

• Along the Main or Broken Diagonals any two numbers separated by \(n/2\) Cells, are a Complementary Pair, i.e. sum 1+\(n^2\).

All the Pandiagonal Squares of Order 4 are Most Perfect. However, when \(n>4\), the proportion Pandiagonal/Most Perfect decreases as \(n\) increases.

4.7 Inlaid Magic Square - Magic Square that contains within itself other Lower Order Magic Squares. The Lower Order Inlaid Magic Squares can be formed by any number inside (unlike a Bordered Magic Square, where the border must contain the lowest and the highest numbers in the series). They can also contain other Inlaid Magic Squares within themselves.

5. Magic Squares I Issue: Souvenir Sheet, Sheetlet, First Day Cover and Stamps

5.1 Souvenir Sheet B152 (1/1): Luo Shu Magic Square

The history of Chinese civilization is full of myths, legends and folk based on mythological beings. Among the first legendary Semi-divine cultural heroes, Fu Xi, Shen Nong (God of Agriculture) and Huang Di or Yellow Emperor, known as “The Three Divine Emperors”, are the most venerated. After Huang Di, followed the “Three Sage Kings” Yao, Shun and Yu, “The Great”, founder of Xia Dynasty. It was during the rein of Yu, “The Great”, that many efforts were put into controlling the effects of great floods.

Among them, “Divine Emperor Fu Xi” and “Sage King Yu”, were, respectively, witnesses of the visits of two mythical creatures: a “Dragon-Horse” and a “Turtle” showing different Dot Patterns on their backs.

Fu Xi, according to the legend, taught his subjects how to fish with nets, to hunt, to domesticate animals and to cook. One day, while he was standing on the banks of Huang He or Yellow River a creature with the form of a “Dragon-Horse” emerged from the river with a Diagram on its back, composed of 55 Dots, in 5 sets. Before submerging, it also left its foot print with 8 Patterns composed of Line Segments. The Diagram became known as He Tu or River Map and the foot print Ba Gua or Eight Trigrams. The Eight Trigrams were later rearranged by King Wen, founder of Zhou Dynasty, which gave origin to the Yi Jing 64 Hexagrams.
Sage King Yu, “The Great”, became legendary ruler for his introduction of a system of flood control, through the construction of dikes and canals. One day, when he was standing on the banks of Luo River, a tributary of the Huang He or Yellow River, a “Turtle” emerged from the water with a Quadrangular Diagram on its shells made of 9 small quadrangular contours with a series of dots inside each one, representing numbers from 1 to 9. King Yu was very surprised to discover that each Row, Column and Main Diagonal of the Quadrangular Diagram contained 15 Dots. The Quadrangular Diagram became know as Luo Shu or Luo River Scroll. It is also commonly called Jiu Gong Shu or Nine Halls Diagram.

The He Tu (River Map) and Luo Shu (Luo River Scroll) are fundamental fabrics in the development of traditional Chinese culture extending its influence to religion, sociology, politics, philosophy, mathematics, medicine, civil engineering, etc.

5.2 Sheetlet

The Sheetlets of I and II issues were conceived to present a disposition for the face values (1 to 9 “patacas”) equal to the disposition that the numbers 1 to 9 occupy in the Luo Shu Magic Square.

In Magic Squares I issue 6 stamps were issued, corresponding to the Superior Row and the Central Row of the Luo Shu Magic Square. The remaining 3 stamps corresponding to the Inferior Row was published in Magic Squares II issue.

With the Magic Squares I and II Sheetlets design, the Macao Philately does not only intend to continue to divulgate and promote scientific knowledge, but also to present to philatelists and Magic Squares enthusiasts a piece that has never been produced in the history of Philately.

Besides, in the Sheetlet several characteristics can be noticed:

• The use of two different colours (black and red) for the odd and even face values, when normally only one colour is indifferently used;

• The filling of the margins with 12 Dudeney Patterns and frequency occurrence of each one of them in the set of 880 different Magic Squares that it is possible to construct for a Natural Magic Square of Order 4.

5.3 First Day Cover ENA174/ENB154: John R. Hendricks – Inlaid Magic Squares

The First Day Cover shows an Inlaid Magic Square of Order 9 with three Inlaid Magic Squares
of Orders 7, 5 and 3. Note that the *Inlaid Magic Square of Order 3* is rotated 45 degrees and is also referred as an *Inlaid Diamond Magic Square*. The numbers used in the *Inlaid Magic Square of Order 9* are from 1 to 81, therefore it is a *Pure Magic Square*. The *Magic Sums* of the *Inlaid Magic Square of Order 9* and its *Lower Order Inlays* are: \(S_9=369\), \(S_7=287\), \(S_5=205\) and \(S_3=123\).

5.4 Stamp S172 (6/1): Sator Palindrome

The *Sator Square* or *Rotas Square* is a *Letters/Words Magic Square* that is composed of a *Latin Palindrome* with the five words – *SATOR AREPO TENET OPERA ROTAS* - that can be read forwards, backwards, upwards and downwards.

The oldest inscription was found in the ruins of Pompeii, which was destroyed in 79 A.D. by Vesuvius eruption of lava and ashes. Others were posteriorly found namely at Corinium (modern Cirencester in England) and Dura-Europos (in modern Syria). There is also a *Sator Square* in the museum at Conimbriga (near Coimbra in Portugal).

The correct translation and its meaning have been under dispute and speculation until the present. A word by word translation can be as follows:

- *Sator* – Sower, seeder, planter, founder, progenitor, originator;
- *Arepo* – Without a clear meaning, probably a proper name (Arepo);
- *Tenet* – To hold, to keep, to possess, to master;
- *Opera* – Work, care, aid, effort, service;

As a sentence, dozens of translations were proposed e.g.:

- “The sower Arepo holds the wheels with effort”;
- “The farmer Arepo keeps the world rolling”;
- “Arepo the farmer holds the works in motion”;
- “The Creator (or Saviour) holds the working of the spheres in his hands”

Some investigators have also speculated that if the five words are properly rearranged, a Greek Cross can be made, that reads horizontally and vertically *PATERNOSTER*, with the remaining letters (A,A and O,O) distributed by each of the 4 quadrants. This translates “OUR FATHER, OUR FATHER” with the letter A and O representing the *Alpha* and *Omega* – the *Beginning* and the *End*. This could make, as yet the speculation goes, the *Sator Magic Square* a safe and hidden way for the early Christians to identify themselves and signal their beliefs to each other without the danger of persecution.
5.5 Stamp S172 (6/2): Franklin – Bent Diagonals

Benjamin Franklin was born in Boston, Massachusetts, 17\textsuperscript{th} January, 1706 and was one of the most influential “Founding Fathers” of the United States, earning the title of “The First American” for his fight for independence. He was also a man of many interests and talents.

In early days he worked as a printer, to become, with the course of the years, a \textit{polymath}, \textit{author}, \textit{politician}, \textit{scientist}, \textit{inventor}, \textit{musician}, \textit{social activist}, \textit{postmaster general}, \textit{statesman} and \textit{diplomat}.

As a \textit{Postmaster}, he was named in 1775 the first United States \textit{Postmaster General}, establishing a postal system that was the basis for the present United States Post Office.

As an \textit{Author} he started to publish the famous \textit{Poor Richard’s Almanack} that became at the time very popular reading. Some of the adages there published remain commons citations even at present.

As an \textit{Inventor} and \textit{Scientist}, among many of his inventions, there are: the \textit{Bifocal Glasses}, the \textit{Lighting Rod}, the \textit{Flexible Urinary Catheter}, and the \textit{Glass Harmonica} etc. He also published several studies about Demography, Atlantic Ocean Currents, Electricity, Meteorology, Cooling Concept, etc.

Being a man with a strong character and clear ethical values, he established for himself, yet very young, as a guide, the following \textbf{13 virtues} that he continued to follow during his life: \textit{Temperance}, \textit{Silence}, \textit{Order}, \textit{Resolution}, \textit{Frugality}, \textit{Industry}, \textit{Sincerity}, \textit{Justice}, \textit{Moderation}, \textit{Cleanliness}, \textit{Tranquility}, \textit{Chastity} and \textit{Humility}.

In addition to the numerous achievements, Benjamin Franklin also left his name associated to the “Magic Squares”. The “Magic Squares” of Benjamin Franklin represented in the stamps shows the same sum for the \textit{Rows} and \textit{Columns} but not for the \textit{Main Diagonals}, i.e, it is only a \textit{Semi Magic Square}. However, it possesses other magic properties as those associated with \textit{Bent Diagonals} either \textit{Continuous} or \textit{Wrap-Around} with sum 260.

In the stamp, several \textbf{Bent-Up-Rows Diagonals} can be seen in different colours, including \textbf{Bent Wrap-Around Diagonals}.

5.6 Stamp S172 (6/3): Dürer – Melencolia I

Albrecht Dürer, son of a goldsmith, was born in 1471, in Nuremberg, Germany. He became
famous as painter, engraver, printmaker, mathematician and academic. He started as an apprentice to Michael Wolgemut when he was young. Later he had been in contact with famous artists like the Schongauer’s brothers, the goldsmiths Caspar and Paul, the painter Ludwig and the sculptor Nikolaus Gerhaert.

Nuremberg was not far away from Venice and Dürer went to Italy twice to study more advance techniques and new artistic expressions. During all these years he could transmit a strong influence and acquire a solid reputation that made him to be regarded as the greatest artist of Northern Renaissance.

After returning to Nuremberg for the second time, he created some famous artistic works as: the paintings, *Adam and Eve* (1507), *The Martyrdom of the Ten Thousand* (1508), *The Virgin with the Iris* (1508), the woodcuts, such as *The First Apocalypses Series* (1498), *The Great Passion* and *The Life of the Virgin* (1511), *The Second Apocalypses Series* (1511), and the well known “Master Prints” (Meisterstiche) *The Knight, The Death and The Devil* (1513), *Saint Jerome In His Study* and *Melencolia I* (1514).

*Melencolia I* is an engraving that includes in the upper right corner, under the bell, a Normal Associated Magic Square of Doubly-Order.

- The two middle *Cells* of the bottom *Row* show the date of the engraving, 1514.
- The *Magic Sum* is $S = 4(4^2+1)/2 = 34$. In addition to the *Rows, Columns* and *Main Diagonals*, the sum $S$ is also possible to be obtained in different ways as follows:
  - The four 2×2 Quadrants, e.g., 16+3+5+10 = 34;
  - The Central Square, e.g., 10+11+6+7 = 34;
  - The Corners of the four 3×3 Squares, e.g., 16+2+9+7 = 34;
  - The Corners of the centered 4×2 and 2×4 Rectangles, e.g., 3+2+15+14 = 34 and 5+8+9+12 = 34;
  - The Corners of the two diagonal 2×3 Rectangles, e.g., 2+8+15+9 = 34 and 5+3+12+14 = 34;
  - The two Skewed Squares, e.g., 8+14+9+3 = 34 and 2+12+15+5 = 34;
  - The Latin Cross Shapes, e.g., 3+5+15+11 = 34 and 2+10+14+8 = 34;
  - The Upside-down Cruciform (St. Peter’s Cross) Shapes 3+9+15+7 = 34 and 2+6+14+12 = 34;
- Any Pair of Cells that are symmetric around the Center sum 17.

**5.7 Stamp S172 (6/4): Su Hui – Xuan Ji Tu – Palindrome**

*Su Hui* (351 A.D.- ?) was a Chinese poetess that lived in *Former Qin* of the *Sixteen Kingdoms* period. She married *Dou Tao*, a government official who later was sent to defend the northern borders. Far away from her husband, she found out that he had taken a concubine. To console her unhappiness
and try to bring him back she composed her Palindrome Poem, Xuan Ji Tu, an array of 29 Lines × 29 Columns, with 841 characters, that can be read at least in 2848 different ways, namely, forward, backward, horizontally, vertically and diagonally. After reading the poem, Dou Tao left his concubine and return to Su Hui, and the love between them became very strong and forever.

This stamp is a square only with 15 Lines × 15 Columns extracted from the central part of the 29 Lines × 29 Columns square that constitutes the full poem Xuan Ji Tu.

Once the poem can be read in so many different ways, for easier understanding on how it can be read, it is necessary to identify some sets of characters, namely:

- The Internal Red Frame, i.e., the Central Red Square (3×3), without the character 心 (xin).
  It is said that this character did not appear in the original poem and was added later by another scholar;
- The Black Frame evolving around the Central Red Square;
- The 4 Black Squares (4×4) at the inner corners of the Peripheral Red Frame;
- The 4 Blue Rectangles (5×4) between the Black Squares;
- The Peripheral Red Frame;
- The Diagonals.

Si Ku Quan Shu (The Imperial Collection of Four) and Shi Yuan Zhen Pin: Xuan Ji Tu by Li Wei are references used for explaining on how to read the poem in different ways.

1. The Internal Red Frame, 8 characters.
   Starting from the middle character 詩(shi) and read it in anti-clockwise with 4 characters for each sentence, we will get two sentences “詩圖瑰璇，始平蘇氏。(shi tu ji xuan，shi ping su shi。)” Starting from the character 蘇(su) to read in the same way, we will get another two sentences “蘇氏詩圖，瑰璇始平。” (su shi shi tu , ji xuan shi ping).

Finally, we can get “詩圖瑰璇，始平蘇氏。蘇氏詩圖，瑰璇始平。” with the meaning of “Xuan Ji Tu is composed by Su Hui who lived in Shi Ping County(始平縣) and it is the origin of Palindrome Poem.”

2. The Black Frame, 16 characters.
   It includes 16 characters in black colour. Starting from the right lower corner 怨義(yuan yi) and read in clockwise with 4 characters for each sentence, we get“怨義興理，辭麗作比，端無終始，詩情明顯。” (yuan yi xing li · ci li zuo bi · duan wu zhong shi · shi qing ming xian ·”) with the meaning of “I use beautiful words and phrases weaved in this brocade to express my ethical
complaints and the rationales of them. However, my love for you continues where you can understand it from the deep emotion meanings embedded in this poem”.

Starting from the left upper corner 端無 (duan wu) and read in clockwise with 5 characters for each sentence, in which the fifth character of last sentence will be repeated as the first character of next sentence. Hence we can get“端無終始詩，詩情明顯怨。怨義興理辯，辭麗作比端。 (duan wu zhong shi shi, shi qing ming xing yuan, yuan yi xing li ci, ci li zuo bi duan)” with the meaning of “I use this poem to express my love to you. The poem contains my obvious discontentment as well. I need to present in the text the rationales behind my ethical complaints. It is because of my love to you that I am using beautiful words and phrases to write this poem.”

Different poems can be extracted starting from different corners and read in 4-characters, 5-characters, clockwise or anti-clockwise etc. It is said that at least 24 poems can be read.

3. The 4 Black Squares (4×4).
   Each square contains 16 characters. Starting from the right upper corner 思感 (si gan) and read it in zigzag way, 4 characters for each sentence. We can get“思感自寧，孜孜傷情。時在君側，夢想勞形。 (si gan zi ning, zi zi shang qing, shi zai jun ce, meng xiang lao xing)” with the meaning of “Thinking of you and the time we passed make me sad and restless. I cannot sleep because I am missing you, and this makes me thin and pallid”.

   The characters of the Black Squares of characters can be read in Row by Row, Column by Column, zigzag in clockwise or anti-clockwise. It is said that at least 176, 4-character poems can be read.

4. The 4 Blue Rectangles (5×4) between the Black Squares.
   Each square contains 20 characters. Take the right rectangle as example and starting from the upper right corner 寒歲 (han su), read in zigzag way with 5 characters for each sentence, we can get “寒歲識凋松，始終知物貞。顔喪改華容，士行別賢仁。 (han su shi diao song, shi zhong zhi wu zhen, yan sang gai hua rong, shi xing bie xian ren)” It means “Pine trees stand firmly in the cold winter. Since you left me, my face grows aging; however, my love for you is eternal just like the pine trees.”

   The characters of the Blue Rectangles can be read in Row by Row, Column by Column, zigzag, in clockwise or anti-clockwise. It is said that at least 176, 5-character poems can be read.

5. The Peripheral Red Frame, 56 characters.
A frame of 56 characters with 8 special Rhyme Characters “欽、林、麟、身、演、沈、神、殷 (qin、lin、lin、shen、shen、chen、shen、yin)” arranged in corners and mid-point of each side.

For example, starting from upper right corner 欽(qin) and read it clockwise till the lower left corner 沙(sha) with 7 characters for each sentence, we can get “欽岑幽巖峻嵯峨，㴱淵重涯網羅，林陽潛曜翳英華，沈浮異逝頹流沙。”(qin cen you yan juan cuo e, shen yuan chong ya jing wang luo, lin yang qian yao yi ying hua, shen fu yi shi tui liu sha。”) with the meaning of “The long curve bank of river, the danger ridge of high mountain, the deep of dark pond, make me fear. I feel depress because my letter to you is lost, just like the warm sunlight for beautiful flowers is blocked by dense forests”.

Starting from different Rhyme Characters and read in different way, it is said that at least 96 7-character poems can be read.

6. The Diagonals, 29 characters each.

There are two Main Diagonals in the brocade. Starting from the 嗟(jie) near the upper right corner, and read it diagonally to the lower left corner with 7 characters for each sentences, we can get “嗟中君容耀多欽，思傷君夢詩璇心。氏辭懷感戚知麟，神輕粲散哀春親。”(jie zhong jun rong yao duo qin, si shang jun meng shi xuan xin, shi ci huai gan qi zhi lin, shen qing can san ai chun qin。”) With the meaning of “Thinking of you make me pale in face, I can only express my love in my poem and meet you in my dream. Although Spring comes, I am still in low spirit and feel sad.” Starting from different corner and read in different way, it is said that at least 96 7-characters can be read.

Su Hui used a lot of Rhyme Characters which are ingenious arranged in the Xuan Ji Tu, and because of this arrangement, even when we start from different character and read in different ways, we still can extract a meaningful poem.

5.8 Stamp S172 (6/5): Lee Sallows – Panmagic 3×3

Born in England in 1944, as a boy Lee Sallows became interested in short wave radio, after which he was to find work as a technician in various branches of the electronics industry. In 1970 he moved to Nijmegen in the Netherlands where he was employed by the Radboud University as an electronics engineer, until his retirement in 2009.

After developing an interest in recreational mathematics, he became an expert on the theory of Magic Squares, a topic to which he contributed several new variations, most notably Alphamagic and
**Geomagic Squares. Sallows has an Erdős number of 2.**

Having become strangely attracted to a **formula due to Édouard Lucas** that characterizes the structure of every 3×3 **Magic Square** (among them the **Luo Shu**), Sallows speculated that it might contain hidden potential.

This speculation was confirmed in 1977 when he published a paper that correlated every **Magic Square of Order 3** with a unique parallelogram on the complex plane. In an improbable move, he then tried substituting the variables in the Lucas formula with geometrical forms, an eccentric notion that led immediately to the invention of **Geomagic Squares**. It turned out to be an unexpected consequence of this find that **Traditional Magic Squares** using numbers **were now revealed as One-dimensional Geomagic Squares**. Other researchers have since then taken notice, among them Peter Cameron who has suggested that “an even deeper structure may lie hidden beyond **Geomagic Squares**”.

The stamp is a **Pandiagonal or Nasik 2-D Magic Square** of Order 3, or one in which, in addition to **Rows and Columns**, all six **Diagonals are Magic**, including the 4 so-called **Broken Diagonals**. In this case the **Magic Target** can also be formed by any three of the four corner **Pieces**. This square is of interest because a **Numerical** equivalent is impossible to construct. The possibility of finding a **Geometrical 3×3 Nasik Square** was thus anything but certain, and their initial discovery an event to celebrate. The resort to disjoint **Pieces** (all of them **Pentominoes**) is an indication of the difficulty encountered in finding it.

**5.9 Stamp S172 (6/6): La Loubère or Siamese Construction Method**

There are several general methodologies to construct **Magic Squares** depending on the class (**Simple, Associated, Pandiagonal**, etc.) and **Order**. However these general methodologies not always apply for all the **Orders** of a certain class, as it is the case for the smallest Orders (3 and 4) because they are special cases. Through the times several methods for constructing Magic Squares have been created namely the following: **La Loubère or Siamese** – **Bachet de Méziriac** – **Philippe de la Hire** – **John Lee Fults** – **Ralph Strachey** – **Stairstep** – **Diagonal** – **Knight’s Move** – **Lozenge** (John Conway) – **Dürer**, etc.

**La Loubère** methodology was created by Simon de la Loubère (1693), a French mathematician that learned it as ambassador to Siam, reason why it is also known by **Siamese**.

**La Loubère** method is one of the most popular to create **Magic Squares** of **Odd Order**. The main characteristic of this method consists in filling the **Cells** of the **Diagonals** in sequential order and moving upward and to the right.
Let’s see how it works:

1. First, the middle Cell of the Row, is filled with number 1;
2. Whenever you reach the top side of the Square, move to the bottom Cell of the right Column and continue to fill the Diagonal upward to the right with the numbers in sequential order;
3. Whenever you reach the right side of the Square, move to the most left Cell of the upper Row and continue to fill the Diagonal as before;
4. Whenever you reach a Cell that is already filled move down one Cell and continue to fill the Diagonal as before;
5. If you reach to upper right corner Cell also move down one Cell and continue as 3.

In this stamp the lines over the Cells show the numeric sequence for filling the Cells according the methodology mentioned at paragraphs 1 to 5.

6. Magic Squares II Issue: Souvenir Sheet, Sheetlet, First Day Cover and Stamps

6.1 Souvenir Sheet B166 (1/1): Method of Knight’s Tour

There are several general methodologies to construct Magic Squares depending on the Class and Order. Among them, the following can be mentioned: La Loubere or Siamese, Bachet de Meziriac, Philippe de la Hire, John Lee Fults, Ralph Strachey, Knight’s Tour, Dürer, etc.

In the Souvenir Sheet of Magic Squares II issue, Method of Knight’s Tour is used to construct a Magic Square of Order 16 with a Closed Circuit or Reentrant.

This method consists, starting in an Initial Cell, to which the number 1 is attributed, to fill numeric and sequentially the Cells, from 1 to \( n^2 \), of a Square of Order \( n \), using the characteristic movements of a Knight Jump, as in the Chess game.

Once the Tour is established, between the Initial Starting Cell and the Final Arriving Cell, and if be possible to proceed it, i.e., to “jump” from the Final Arriving Cell to the Initial Starting Cell with a legal Knight movement, the Tour is said Closed or Reentrant and, in this case, the Initial Starting Cell can be anyone. On the contrary, the Tour is said Open or Non-reentrant.

When the Knight Jump establish a Tour that generates a true Magic Square, i.e., when the Rows, Columns, and Main Diagonals add up the same Magic Sum, it is said that the Tour is a Magic Knight Tour (MKT).
When the Main Diagonals Sum is different from the Rows and Columns Sum, the Tour is said a Semi Magic Knight Tour (SMKT).

The history to try, by the Knight Tour Method to reach all the Cells of the Chess Board, $8 \times 8$, or Boards with different Dimensions, $n \times n$ or $m \times n$, in only one Tour, comes from Antiquity, while the tentative to create Magic Squares is very much recent, although it is possible to mention several tentatives, among them, by famous mathematicians like Abraham De Moivre (1667-1754), Leonhard Euler (1707-1783) and Adrien-Marie Legendre (1752-1833).

To De Moivre is attributed the prowess to be the first to establish a Tour, in 1722, although Open, able to touch all Chess Board Cells, $8 \times 8$. Years later, the same endeavor, but in a more complex way, since the Tour is Closed, could be achieved by Euler and Legendre, although not Magic. Euler was among the first to study Knight Tours systematically in a scientific way, around 1759. He also created one of the first methods for finding them. However, the best-known historical procedure (Warnsdorff's Rule) was created by H.C. Warnsdorff, a German mathematician, in 1823.

Regarding Semi Magic Squares Knight Tours, others, as: William Beverly, who was the first to publish, in 1847, a Semi Magic Square of Order 8 with an Open Tour: Carl Wenzelides who published, in 1849, a Semi Magic Square of Order 8 with a Closed Tour: Krishnaraj Wadiar, who published, in 1852, a Semi Magic Square of Order 8 with a Closed Tour: Carl F. Jaenisch who published, in 1859, a Semi Magic Square of Order 8 with a Closed Tour: M. A. Feisthamel who published, in 1884, a Magic Square of Order 8 with an Open Tour: are distinguished in the efforts to create, for the Order 8 or, the same to say, for the Chess Board, a Magic Square with a Closed Tour. This objective comes to be proved impossible to fulfill in August 2003 (Guentar Stertenbrink 2003) through the complete computational enumeration of all possibilities. However, during the process 140 different Semi Magic Tours were discovered.

The interest aroused by the creation of Magic Squares using the Knight Tour Method in different dimension Boards, led to studies that concluded not to be possible to exist a Magic Square Tour in Boards $n \times n$, with $n$ Odd, although it is feasible for Boards of Order $4k \times 4k$, with $k>2$.

Among these are to be mentioned the first 4 Magic Squares of Order 12, although with an Open Tour, created by Awani Kumar in 2003 and published in the Games and Puzzles Journal Issue 26. Without answer yet remains the question about the existence or not of Magic Squares of Order 12 with a Closed Tour.

Finally, is introduced the Magic Square of Order 16, with a Closed Tour, the Knight can jump from Cell 256 to 1, that constitutes the design of the Souvenir Sheet, published by the author Joseph

As can be verified in the Souvenir Sheet the attribution of the numbers 1 to 256 in the Cells is made sequentially and respects the Knight Jump rule on the game of Chess. The Magic Sum is 2056.

6.2 Sheetlet

As occurred in Magic Squares I, the Sheetlet presents a disposition for the face values of the stamps (1 to 9 “patacas”) equal to the disposition that the numbers 1 to 9 occupy in the Luo Shu Magic Square.

In this issue, the last three stamps, with the face values of 8, 1 and 6 “patacas”, are issued, corresponding to the Inferior Row, as mentioned in the previous Introductory Note.

On the Lateral Margins two Magic Squares Tiling Schemes proposed by David Harper are shown, which are based in the correspondent between the decimal and binary numerical bases.

Considering that it is a Configuration with 16 motifs (1 to 16) it is suitable to Tile Order 4 Magic Squares and their continuous repetitions.

On the Right Margin, the Scheme takes as the basis a Right Triangle, while on the Left Margin a Square, both with an area equal to 1/4 of the Cell area.

When, in an Order 4 Magic Square and their extensions, the numbers 1 to 16 are substituted by the equivalent Tiles, Patterns of great beauty can be obtained.

6.3 First Day Cover ENA195/ENB168: Yang Hui Magic Circles

Not too much is known about ancient Chinese Mathematicians, because under the instructions of Qin Shi Huang (秦始皇) (259-210 BC), not only some books were burnt but also many scholars were killed (213 BC).

Among the Mathematical Classic works, Jiu Zhang Suan Shu (九章算術), Nine Chapters of the Mathematical Art (10 BC-2 ) is probably the greatest. It is composed, as the title suggests, by 9 Chapters with 246 problems covering practical life aspects as: Weights, Measures, Surveying, Tax Collection, etc. and Linear Equations.

It was only during the Tang Dynasty (618-907) that the most important mathematical works, until then known, were compiled (656), latter known as Suan Jing Shi Shu (算經十書), the Ten
Computational Canons.

The XIII Century was probably one of the most relevant periods in the History of Chinese Mathematics, with the publication of *Shu Shu Jiu Zhang (數書九章)*, *Mathematical Treatise in Nine Sections*, 1247, by *Qin Jiu Shao (秦九韶)* and *Ce Yuan Hai Jing (測圓海鏡)*, *Sea Mirror of Circle Measurements*, by *Li Ye (李冶)*, followed 15 years later, by the works of *Yang Hui (楊輝)*.

*Yang Hui (楊輝)*, (1238-1298), born in Qiantang (錢塘) (modern Hangzhou (杭州)), Zhejiang Province (浙江省), during the late Song Dynasty (宋朝) (960-1279), learned mathematics from the works of *Liu Yi (劉益)* who was a native of Zhongshan in Hebei Province (河北中山府).

Among his works, it is relevant to mention the followings:


Subsequently, these last three series of *Yang Hui’s* works, consisting of 7 volumes, were later assembled and published (1378) in what is now known by *Yang Hui Suanfa (楊輝算法)*, *Yang Hui’s Methods of Computation*.

The topics covered by *Yang Hui* include Multiplication, Division, Root-extraction, Quadratic and System Equations, Series, Computations of areas of Polygons as well as Magic Squares, Magic Circles, the Binomial Theorem and, the best known work, his contribution to the *Yang Hui’s Triangle*, (discovered by his predecessor *Jia Xian (賈憲)*), four hundred years later rediscover by the French Mathematician *Blaise Pascal* (1653).

At the Bottom Left Corner of the *First Day Cover* is presented the *Yang Hui Magic Circles*.
These Nine Circles are composed by 72 Numbers, from 1 to 72, having each individual Circle 8 Numbers. The neighbouring Numbers make Four Additional Circles, also with 8 Numbers each, thus making altogether 13 Circles in the Square [NW, N, NE, (NW, N, C,W), (N, NE, E, C), W, C, E, (W, C, S, SW), (C, E, SE, S), SW, S, SE], with 8 numbers each, and the following proprieties:

- Total Sum of the 72 Numbers = 2628;
- Sum of the 8 Numbers in each Circle = 292;
- Sum of the 3 Circles along the Horizontal Lines = 876;
- Sum of the 3 Circles along the Vertical Lines = 876;
- Sum of the 3 Circles along the Main Diagonals = 876.

6.4 Stamp S193 (3/1): McClintock / Ollerenshaw – Most Perfect

It is not possible to establish the history of Most-Perfect Magic Squares without to mention Kathleen Timpson Ollerenshaw. Despite to be almost completely deaf from early age she could earn a DPhil in Mathematics from Oxford University. Although much of her adult life was devoted to voluntary social services, public education and politics - she was made Dame Commander of the Order of British Empire (DBE) - she also could take some time to work on Mathematics.

Not only she published a paper in 1980 where she explained one of the first general methods for solving the Rubik Cube Puzzle, but also, in 1982, with Hermann Bondi, they developed a mathematical analytical construction that could verify the number 880 for the essentially different Magic Squares of Order 4 proposed by Bernard Frénicle de Bessey in XVII Century.

After this achievement she began to study Pandiagonal Magic Squares based on works published by Emory McClintock in 1897. After several years, in 1986, Kathleen Ollerenshaw published a paper where, making use of Symmetries, she proved that there are 368,640 essential different Most-Perfect Magic Squares of Order 8.

Step by step, she could discover how to construct and how to count the total number of Most-Perfect Magic Squares of Order Power 2, then for Magic Squares whose Order is a Multiple of Power 2, and finally, for all with an Order Multiple of 4.

Together with David Brée, who helps her to organize her research notes and proof-reading, they finally, published in 1998 the book “Most-Perfect Pandiagonal Magic Squares: Their Construction and Enumeration”. The book received international recognition and has constituted a remarkable accomplishment for a woman of age 85.
Later she said: “I hope to encourage others. The delight of discovery is not a privilege reserved solely for the young.”

As it was previously defined, a *Most-Perfect Magic Square* is a *Pandiagonal Magic Square* of *Doubly-Even Order*, with the additional two proprieties:

- The *Cells* of any square of Order 2, \((2 \times 2 \text{ Cells})\) extracted from it, including *Wrap-Around* sum the same constant value, \(2(1 + n^2)\);
- Along the *Main* or *Broken Diagonals*, any two numbers separated by \(n/2\) *Cells*, are a *Complementary Pair*, i.e. *Sum* \(1+n^2\).

In the case of the *Most-Perfect Magic Square of Order 8* reproduced in the stamp the mentioned properties show the following results:

- \(2(1 + n^2) = 2(1 + 8^2) = 130\)
  examples: \((59 + 38 + 7 + 26) = (48 + 33 + 18 + 31) = (41 + 4 + 32 + 53)\) include *Wrap-Around* = 130

- \((1 + n^2) = (1 + 8^2) = 65\)
  examples: \((1 + 64) = (34 + 31) = (25 + 40) = (35 + 30)\) *Broken Diagonal* = 65

6.5 Stamp S193 (3/2): David Collison – Patchwork

**David M. Collison** (1937-1991) was born in United Kingdom and lived in Anaheim, California. He was a fruitful creator of *Magic Squares* and *Cubes* to whom, not only is attributed the creation of the *Patchwork Magic Square* presented in this stamp, but also the creation of a *Bimagic Cube of Order 25*, published later by **John R. Hendricks**. He specialized in *Generalized Shapes* from which he created the *Patchwork Magic Squares*.

A *Patchwork Magic Square* is an *Inlaid Magic Square* – one *Magic Square* that contains within it others *Magic Squares*, often placed in the *Quadrants* – that contain *Magic Squares* or *Odd Magic Shapes* within it. The most common *Shape* is *Magic Rectangle*, but *Diamond*, *Cross*, *Elbow* and *L Shapes* can also be found.

These *Shapes* are *Magic* if the *Sum* in each *Direction* is proportional to the number of *Cells*. For example, if a *6 \times 8 Rectangle* has a *Sum* of 120 in the *Short Direction*, the *Sum* in *Long Direction* should be 160. *Main Diagonals* are not required to be the *Magic Sum* unless they belong to a *Square*. 

22/25
The Patchwork Magic Square of Order 14 reproduced in this stamp has the following properties:

- Contain: Four **Order 4 Magic Squares**, $4 \times 4$, in the **Quadrants**; one **Magic Cross**, $6 \times 6$, in the **Centre**; four **Magic Tees**, $6 \times 4$, on the **Centre Sides**; and four **Magic Elbows**, $4 \times 4$, in the **Corners**.
- All the **Shapes** sum to a **Constant** directly proportional to the number of **Cells** in a **Row**, **Column** or **Diagonal**: $S_2=197$; $S_4=394$; $S_6=591$; $S_{14}=1379$. The **Mean** for each **Cell** is **98.5**.

Harvey Heinz, transformed the Four **Magic Squares of Order 4** mentioned above, from **Associated** to **Pandiagonal**, by moving two **Bottom Rows** to the **Top** and then two **Left Columns** to the **Right**.

**6.6 Stamp S193 (3/3): Inder Taneja – IXOHXI 88**

**Inder Taneja**, Professor at the Department of Mathematics, University of Santa Catarina, Brazil, 1978-2012. M.Sc. (1972) and Ph.D. (1975) degrees in Mathematics from Delhi University, India.

Post-doctoral research in Italy (1983) and Spain (1989). Research interests in **Information Theory** especially on Information Measures, Probability of Error, Noiseless Coding, Fuzzy Set Theory, Inequalities, etc.

Recent interest are on **Magic Squares** and **Numbers** Applications and Information Measures to Genetic Code, DNA, etc. Published more than 100 research papers in journals of international reputations. Five chapters in information measures in book series. One online book on Information Measures.

**IXOHXI Magic Squares** are a special series that can not only show common properties like other **Magic Squares** and still include alternative properties as **Symmetries**, **Rotations** and **Reflections**.

The word **IXOHXI** is itself a **Palindrome** and **Symmetric (Reflection)**, in relation to the “H” centre.

As it can be seen in the stamp, the **Symmetric Properties** not only apply to the Square itself, but also to the numbers of the **7 Segments LED Display** that have been chosen intentionally to fill the **Cells**.
From the 10 digits created with a 7 Segments LED Display, only 0, 1, 2, 5 and 8, written in digital format, remain the same after a **180 Degree Rotation** (6 becomes 9 and 9 becomes 6). Looking from a **mirror**, these five digits remains the same, with the change that 2 becomes 5 and 5 becomes 2.

It is yet to mention that the 4 digits (0, 1, 2 and 5) used to construct the **Magic Square of Order 4**, are precisely the same digits that constituted the year **2015**, year of its publication as a stamp.

Taking into consideration the 5 digits and their Symmetric Properties, Inder Taneja created the **IXOHOXI Universal 88 Magic Square** reproduced in this stamp with the following properties:

1. The Magic Square still remains a Magic Square:
   - After a Rotation of **180 degree**;
   - If it is seen in a mirror, or reflected in water or seen from the back of the sheet;
   - The Magic Sum \( S \) of the Magic Square of Order 4 is equal to **88**, number that also enjoys Symmetrical properties.

2. Additionally working with a Magic Square of order \( 5 \times 5 \) using the digits 0, 1, 2, 5 and 8 forms a 176 (88+88). In this case the Magic Square “IXOHOXI Universal 88+88” is Pandiagonal.

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