MAGIC SQUARES

I and II

Contents

1. Introductory Note

By dedicating the 6th and 7th issues of the **Science and Technology** series to **Magic Squares**, Macao Post intends not only to promote this theme in **scientific** and **cultural terms**, but to create also a **unique product** in the history of Philately, i.e. a *Magic Square* formed by stamps specifically produced for this purpose. The issue **Magic Squares I** was published on 9th October 2014, World Post Day, whereas the issue **Magic Squares II** was published on 12th of November 2015.

In the *Magic Squares I* issue, besides the **Souvenir Sheet** based on the **Luo Shu Magic Square**, it was issued a **Sheetlet** with the Shape of a **Matrix** with **3 Rows × 3 Columns** (9 stamps), a *Luo Shu Magic Square* replica, composed of six stamps: three occupying the **Superior Row**, with the face values of 4, 9 and 2 "patacas" and the remaining three, occupying the **Central Row**, with the face values of 3, 5 and 7 "patacas". The **Inferior Row** was left intentionally empty to accommodate the remaining three stamps to be published in the *Magic Squares II* issue.

In the *Magic Squares II* issue, it was also issued a *Sheetlet* with the Shape of a *Matrix* with *3 Rows* **×** *3 Columns* (9 stamps), a *Luo Shu Magic Square* replica, composed of three stamps with the face values of 8, 1 and 6 "patacas" occupying the *Inferior Row*. The *Superior* and *Central Rows* were left intentionally empty to accommodate the six stamps published in the *Magic Squares I* issue.

Therefore, it is possible to build, taking as basis the first issue Sheetlet (adding to it the last three stamps issued) or the second issue Sheetlet (adding to it the first six stamps issued), a Luo Shu Magic Square made of stamps.

The interest for *Magic Squares* is transversal in Chinese and Western Cultures. The importance of this theme in Chinese culture can be traced back to ancient times of its history which is shown by the myths illustrated in the **Luo Shu Souvenir Sheet** of *Magic Squares I* issue, as well as the **Magic Circles** created by **Yang Hui** presented in the **First Day Cover** of *Magic Squares II* issue.

2. Brief Early History

According to some researchers and academics the early history of *Magic Squares* is not well researched neither documented. However, there is a general view that three civilizations have contributed to its creation: the **Chinese**, the **Indian** and the **Arab**.

The first recorded *Magic Square*, an *Order 3*, is known as **Luo Shu** or **Luo River Scroll**, was presented in detail in the *Souvenir Sheet* of *Magic Squares I* issue. Later, *Magic Squares* and **Magic**

Circles appeared in Chinese Literature around the Northern Song Dynasty (960-1127).

Some researchers thought that Magic Squares travelled from China to India, where several *Magic Squares of Order 4* were presented by the Buddhist Philosopher **Nagarjuna**, who lived about the Second Century, in his work **Kaksaputa**. It took more than 1000 years to find a **Jaina Inscription** that showed, again, an *Order 4 Magic Square*, although with special properties not yet present in the works of his predecessor. This kind of *Magic Squares* was named lately **Jaina Magic Squares**.

However, it is due to the Mathematician **Narayana Pandit** a systematic study of Magic Squares that, in his work **Ganita Kaumudi** (1356), presented general methods for the construction of several sorts.

From India, probably, *Magic Squares* travelled to Arabic world and it is known that Islamic and Arabic Mathematicians took awareness of *Magic Squares* before the Seventh Century. So, it was not surprise that *Magic Squares of Order 5 and 6* appear in **Rasa'il Ikhwan al-Safa**, **Encyclopedia of the Brethren of Purity**, composed by **52 Epistles** in Mathematics, Natural Sciences, Psychology and Theology, in Baghdad, around the year 983. Later, around 1200, **Ahmad al-Buni**, a famous Arab Mathematician, also studied *Magic Squares*.

Arabs do not only use *Magic Squares* in making astrological calculations and predictions but also, like *Ahmad al-Buni*, believe in their mystical properties.

The introduction of *Magic Squares* in Europe is believed to take place, not only, in 1280, through a **Spanish Manuscript** where to each one *Magic Square* a **Planet** was associated, as in the **Islamic Literature**, as well as, around year 1300, when the Greek Byzantine, **Manuel Moschopoulos** (1265-1316) wrote a mathematical treatise based on the works of *Ahmad al-Buni*. *Magic Squares* of *Order 6* and *9* appeared again in the XIV Century, in Florence, in a manuscript of the (**Trattato d'Abbaco**) **Treatise of the Abacus** by **Paolo Dagomari**, Mathematician, Astronomer and Astrologer. Following *Moschopoulos*, **Luca Pacioli** (1445-1517), fellow Florentine like *Dagomari*, also studied *Magic Squares* and presented in his work **De Viribus Quantitatis**, around 1500, several of *Order 3* to *9*.

Following these pioneers, during XV to XVII Centuries, the most notable names in working or developing **Methods** for solving *Magic Squares* were, namely: **Albrecht Dürer** (1471-1528), whose engraving Melancholia I, an *Order 4 Magic Square*, was also introduced in *Magic Squares I* issue；the German Physician and Theologian **Heinrich Cornelius Agrippa** (1486-1535), who wrote in 1510 "**De Occulta Philosophia, Libri Tres**", at that time one of the most influential books around Europe in which he exposed, with examples, the mystical powers of *Magic Squares* of *Orders 3 to 9*；**Johann**

Faulhaber (1580-1635)；**Claude Gaspar Bachet de Meziriac** (1581-1638)；**Simon de la Loubere** (1642-1729) whose Method of Construction of *Magic Squares* was introduced in a stamp in *Magic Squares I* issue.

Over the years several mathematicians as **Pierre de Fermat** (1601-1665), **Bernard Frenicle de Bessy** (1605-1675), **Benjamin Franklin** (1706-1790) presented in the stamp **Bent Diagonals** in the *Magic Squares I* issue, **Leonhard Euler** (1707-1783), **Arthur Cayley** (1821-1895) etc. have shown interest to study *Magic Squares* discovering interesting relations and properties.

3. Magic Squares: Basic Definitions

A – Numeric Magic Squares

3.01 **Cell** – Each one of $n^2 = n \times n$ small squares that constitute the *n Rows* and *n Columns* of the square. *Cells* can be filled with **Numbers**, **Letters**/**Characters** or **Geometric Shapes**/**Pieces**.

3.02 **Closed Magic Line** – Lines that connect the centers of Cells in numeric sequence, including the return to the first, e.g. 1, 2, 3, …, n^2,…3, 2, 1 for a Normal Magic Square (after Claude Bragdon). When the areas between the Magic Line are filled with different colours, some very interesting patterns may be created, called Sequence Patterns (after Jim Moran).

3.03 **Column** – Set of *n* vertical Cells. A square of Order *n* has *n* Columns.

3.04 **Complementary Pair** or **Complementary Numbers** – Pair of Cells or Pair of Numbers which sum is equal to the sum of the first and last terms of the series, e.g. $1+n^2$ for a Normal Magic Square.

3.05 **Diagonals**

3.05.1 **Bent Diagonal** – Diagonal, with the same *n* number of *Cells* as the *Order n* of the Square, that:

- Starts at any edge *Cell* of the Square and finish at the opposite edge *Cell*;
- Is composed by two V-shape perpendicular and symmetric segments with the vertice coinciding with: the center of the Square, for *Even Order*; the center of the central *Cell*, for *Odd Order*.

There are $4 \times n$ *Bent Diagonals*, distributed as follows:

- *Even Order*: Continuous, 2(*n*+2), *Wrap-Around*, 2(*n*-2);

- *Odd Order*: Continuous, 2(n+1), *Wrap-Around*, 2(n-1).
  Bent Diagonals also sum the *Magic Sum S* and are the main characteristic of **Benjamin Franklin** "*Magic Squares*".

3.05.2 **Broken Diagonal Pair –** Two *Short Diagonals* – with the same or different number of *Cells* – that are parallel to a *Main Diagonal*, but on opposite sides, and when connected, contain the same number of *Cells*, as each *Row*, *Column* or *Main Diagonal*, i.e. the *Order n* of the *Magic Square*.
Broken Diagonals also sum the *Magic Sum S* and are the main characteristic of the **Pandiagonal Magic Squares**.

3.05.3 **Main Diagonal –** Each one of the two diagonals, called **Leading** or **Left Diagonal** and **Right Diagonal**, constituted with n *Cells*, which connect the opposite corners of the square.

3.05.4 **Opposite Short Diagonals Pair** – Two *Short Diagonals* which are parallel and on opposite sides of a *Main Diagonal* and contain the same number of *Cells*.

3.05.5 **Pandiagonal –** That include all diagonals: *Main Diagonals* and *Broken Diagonals*.

3.05.6 **Short Diagonal –** Diagonal that is parallel to one of the *Main Diagonals* and intercept two adjacent sides of the square. A *Short Diagonal* may have 1 to n-1 Cells.

3.05.7 **Wrap-Around Diagonal –** *Bent Diagonal* obtained through *Wrap-Around* process.

3.06 **Magic Sum** or **Magic Number** or **Magic Constant –** The constant sum of the *n Cells* of each *Row*, *Column*, *Main Diagonal*, etc. For a *Normal Magic Square* the *Magic Sum S* can be calculated by the formula $S = n(n^2+1)/2$, where n is the *Order of the Magic Square*.

3.07 **Normal** or **Pure** or **Traditional Magic Square of Order n –** *Magic Square* where the numbers used for filling the n^2 *Cells* are consecutive positive integers from 1 to n^2.

3.08 **Order n –** There are four classes of *Magic Squares* according the *Order*:

- **Even –** When n is even, i.e. n = 2, 4, 6, 8…;
- **Odd –** When n is odd, i.e. n = 3, 5, 7…;
- **Doubly–Even –** When n is multiple of 4, i.e. n = 4, 8, 12…;
- **Singly** or **Oddly–Even –** When n is even but is not divisible by 4, i.e. n = 6, 10, 14….

3.09 **Row –** Set of *n* horizontal *Cells*. A *Square of Order n* has *n Rows*.

3.10 **Square of Order n –** Square composed with *n Rows, n Columns* and n^2 *Cells*.

3.11 **Symmetrical Pair** or **Symmetrical Cells –** Pair of *Cells* diametrically equidistant or symmetric in relation to the Center of the Square.

3.12 **Wrap-Around –** To connect the square opposite sides (i.e. left-right or up-down) in order to make a continuous cylindrical surface where the opposite sides overlay.

B – Letters/Words/Characters Magic Squares

3.13 **Palindrome –** A number, word or phrase that can be read the same in different directions (e.g. right-to-left or left-to-right). However, as it is the case for the S*u Hui Xuan Ji Tu Palindrome* poem, the verses, in Chinese language, can also be read in different directions, not with the same meaning, but yet meaningful.

C – Geometric Magic Squares

3.14 **Geometric Magic Square** or **Geomagic Square –** An array of n^2 Cells (*n Rows* × *n Columns*) each occupied by a distinct geometrical (usually planar) **Shape** or **Piece**, such that *n* of them taken from any *Row*, *Column* or *Main Diagonal* can be assembled to create a *Larger Constant Shape* or *Piece* known as the *Magic Target*. By the **Dimension** of a *Geomagic Square* is meant the dimension of its *Shapes* or *Pieces*.

3.15 **Magic Target –** The **Larger Constant Shape** or **Piece** formed by the union of the *n Shapes* or *Pieces* occurring in any *Row*, *Column*, or *Main Diagonal*. The *Shapes* or *Pieces* used are usually planar, but may be of any dimension. 3-D *Pieces* will thus assemble to form 3-D *Targets*, while 1-D *Shapes* are simply straight line segments (each of which could alternatively be represented by a single number). The *Magic Target* will then be another straight line (equal in length to the sum of those numbers). From this is seen that **Numerical Magic Squares are in fact a particular kind of Geomagic Squares**.

4. Magic Squares: General Classification

The *Magic Squares* can be categorized, according to special properties they may have, in different ways. When the *Cells* of the square are filled with *Numbers*, *Letters*/*Characters* or *Geometric Shapes*, the square is called, respectively, **Numeric Square**, **Letters/Words/Characters Square** and **Geomagic Square** (after Lee Sallows).

One of the most frequent classifications divides them in three classes: *Simple, Associated* and *Pandiagonal*. However, removing or adding them some properties, we can also consider in addition the following three: *Semi-Magic*, *Semi-Pandiagonal* and *Most Perfect*.

When they show special characteristics, take others names as: **Bimagic**, **Bordered** or **Concentric**, **Inlaid, Alfamagic**, **Latino**, **Domino**, **IXOHOXI**, **Prime**, **Serrated**, etc.

The previous six classes are defined, for *Magic Squares of Order n,* in the following paragraphs 4.1 to 4.6, **according the increasing complexity of their properties**.

4.1 **Semi-Magic Square –** Array of n^2 *Cells* (*n Rows* × *n Columns*) where the sum of each *Row* and of each *Column* is equal to the *Magic Sum S*. The sum of one or the two *Main Diagonals* is different of *S* and hence the name *Semi-Magic*.

4.2 **Simple**, **Normal**, **Numeric Magic Square of Order n** or simply **Magic Square –** Array of n^2 *Cells* (*n Rows* × *n Columns*) each one filled with a certain number in a way that the sum of each *Row*, each *Column* and the two *Main Diagonals* all have the same value *S*, called *Magic Sum*.
The fulfillment of the above mentioned properties is the minimum requirement to qualify as a *Magic Square.*

4.3 **Associated** or **Regular** or **Symmetrical Magic Square –** *Magic Square* where the sum of all Symmetric Pairs is equal to the sum of the first and last terms of the series, i.e. $1 + n^2$.
In an *Odd Order Associated Magic Square*, the *Center Cell* is always equal to the middle number of the series, i.e. $(1+n^2) / 2$.
No *Singly-Even Order Associated Magic Square* exists.

4.4 **Semi-Pandiagonal** or **Semi-Diabolic** or **Semi-Nasik Magic Square –** *Magic Square* with the following properties:
Even Order:

- The sum of an *Opposite Short Diagonal Pair* with *n Cells*, is equal to the *Magic Sum, S*.

Odd Order:

- The sum an *Opposite Short Diagonal Pair* with *n*-1 *Cells* plus the *Center Cell,* is equal to the *Magic Sum, S*;
- The sum of an *Opposite Short Diagonal Pair* with *n*+1 *Cells* minus the *Center Cell*, is equal to the *Magic Sum, S*

For a *Magic Square* of *Order n*, the *Magic Sum S*, is always calculated taken into account the *n Cells* of the *Rows, Columns* and *Pandiagonal Diagonals.* This is the reason why it is necessary to add or subtract the *Center Square Cell* to the *Opposite Short Diagonal Pair Cells*, when these are, respectively, *n-1* and *n+1*.

4.5 **Pandiagonal** or **Diabolic** or **Nasik** or **Continuous Magic Square** - A *Magic Square* where each *Broken Diagonal Pair* sum is equal to the *Magic Sum*, *S*.
The *Pandiagonal Magic Square* is considered **one of the most sophisticated** among the classes of *Magic Squares*;
No *Singly-Even Order Pandiagonal Magic Square* exists.

4.6 **Most Perfect Magic Square** - *Pandiagonal Magic Square* of *Doubly-Even Order*, with the two additional properties:

- The *Cells* of any square of *Order 2*, (2 × 2 *Cells*) extracted from it, including *Wrap-Around*, sum the same value, 2 $(1+n^2)$;
- Along the *Main* or *Broken Diagonals* any two numbers separated by *n*/2 *Cells*, are a *Complementary Pair*, i.e. sum $1+n^2$.

All the *Pandiagonal Squares* of *Order 4* are *Most Perfect*. However, when $n>4$, the proportion Pandiagonal/Most Perfect decreases as *n* increases.

4.7 **Inlaid Magic Square** - *Magic Square* that contains within itself other **Lower Order Magic Squares**. The *Lower Order Inlaid Magic Squares* can be formed by any number inside (unlike a **Bordered Magic Square**, where the border must contain the lowest and the highest numbers in the series). They can also contain other *Inlaid Magic Squares* within themselves.

5. Magic Squares I Issue: Souvenir Sheet, Sheetlet, First Day Cover and Stamps

5.1 Souvenir Sheet B152 (1/1): Luo Shu Magic Square

The history of Chinese civilization is full of myths, legends and folk based on mythological beings. Among the first legendary Semi-divine cultural heroes, **Fu Xi**, **Shen Nong** (God of Agriculture) and **Huang Di** or **Yellow Emperor**, known as “**The Three Divine Emperors**”, are the most venerated. After *Huang Di*, followed the “**Three Sage Kings**” **Yao**, **Shun** and **Yu**, “**The Great**”, founder of Xia Dynasty. It was during the rein of *Yu*, “*The Great*”, that many efforts were put into controlling the effects of great floods.

Among them, “*Divine Emperor Fu Xi*” and “*Sage King Yu*”, were, respectively, witnesses of the visits of two mythical creatures: a “**Dragon-Horse**” and a “**Turtle**” showing different **Dot Patterns** on their backs.

Fu Xi, according to the legend, taught his subjects how to fish with nets, to hunt, to domesticate animals and to cook. One day, while he was standing on the banks of **Huang He** or **Yellow River** a creature with the form of a “*Dragon-Horse*” emerged from the river with a **Diagram** on its back, composed of **55 Dots**, in **5 sets**. Before submerging, it also left its foot print with **8 Patterns** composed of **Line Segments**. The *Diagram* became known as **He Tu** or **River Map** and the foot print **Ba Gua** or **Eight Trigrams**. The *Eight Trigrams* were later rearranged by King **Wen**, founder of Zhou Dynasty, which gave origin to the **Yi Jing 64 Hexagrams**.

Sage King Yu, "*The Great*", became legendary ruler for his introduction of a system of flood control, through the construction of dikes and canals. One day, when he was standing on the banks of **Luo River**, a tributary of the *Huang He* or *Yellow River*, a *"Turtle"* emerged from the water with a **Quadrangular Diagram** on its shells made of 9 small quadrangular contours with a series of dots inside each one, representing numbers from 1 to 9. *King Yu* was very surprised to discover that each *Row*, *Column* and *Main Diagonal* of the *Quadrangular Diagram* contained **15 Dots**. The *Quadrangular Diagram* became know as **Luo Shu** or **Luo River Scroll**. It is also commonly called **Jiu Gong Shu** or **Nine Halls Diagram**.

The *He Tu* (*River Map*) and *Luo Shu* (*Luo River Scroll*) are fundamental fabrics in the development of traditional Chinese culture extending its influence to religion, sociology, politics, philosophy, mathematics, medicine, civil engineering, etc.

5.2 Sheetlet

The **Sheetlets** of I and II issues were conceived to present a disposition for the face values (1 to 9 "patacas") equal to the disposition that the numbers 1 to 9 occupy in the *Luo Shu Magic Square*.

In *Magic Squares I* issue 6 stamps were issued, corresponding to the **Superior Row** and the **Central Row** of the *Luo Shu Magic Square*. The remaining 3 stamps corresponding to the **Inferior Row** was published in *Magic Squares II* issue.

With the *Magic Squares I* and *II Sheetlets* design, the Macao Philately does not only **intend to continue to divulgate and promote scientific knowledge**, but also to present to philatelists and Magic Squares enthusiasts **a piece that has never been produced in the history of Philately**.

Besides, in the Sheetlet several characteristics can be noticed:

- The use of **two different colours** (**black** and **red**) for the **odd** and **even face values**, when normally only one colour is indifferently used;
- The filling of the margins with **12 Dudeney Patterns** and frequency occurrence of each one of them in the set of **880 different** *Magic Squares* that it is possible to construct for a *Natural Magic Square of Order 4*.

5.3 First Day Cover ENA174/ENB154: John R. Hendricks – Inlaid Magic Squares

The First Day Cover shows an **Inlaid Magic Square of Order 9** with three *Inlaid Magic Squares* *of Orders 7, 5* and *3*. Note that the *Inlaid Magic Square of Order 3* is rotated 45 degrees and is also referred as an **Inlaid Diamond Magic Square**. The numbers used in the *Inlaid Magic Square of Order 9* are from 1 to 81, therefore it is a **Pure Magic Square**. The **Magic Sums** of the *Inlaid Magic Square of Order 9* and its *Lower Order Inlays* are: S_9=369, S_7=287, S_5=205 and S_3=123.

5.4 Stamp S172 (6/1): Sator Palindrome

The **Sator Square** or **Rotas Square** is a *Letters/Words Magic Square* that is composed of a *Latin Palindrome* with the five words – **SATOR AREPO TENET OPERA ROTAS** - that can be read forwards, backwards, upwards and downwards.

The oldest inscription was found in the ruins of Pompeii, which was destroyed in 79 A.D. by Vesuvius eruption of lava and ashes. Others were posteriorly found namely at Corinium (modern Cirencester in England) and Dura-Europos (in modern Syria). There is also a *Sator Square* in the museum at Conimbriga (near Coimbra in Portugal).

The correct translation and its meaning have been under dispute and speculation until the present. A word by word translation can be as follows:

Sator – Sower, seeder, planter, founder, progenitor, originator;
Arepo – Without a clear meaning, probably a proper name (Arepo);
Tenet – To hold, to keep, to possess, to master;
Opera – Work, care, aid, effort, service;
Rotas – Wheel, rotate.

As a sentence, dozens of translations were proposed e.g.:

- “The sower Arepo holds the wheels with effort”;
- “ The farmer Arepo keeps the world rolling”;
- “Arepo the farmer holds the works in motion”;
- **“The Creator (or Saviour) holds the working of the spheres in his hands”**

Some investigators have also speculated that if the five words are properly rearranged, a Greek Cross can be made, that reads horizontally and vertically **PATERNOSTER**, with the remaining letters (**A,A** and **O,O**) distributed by each of the 4 quadrants. This translates “**OUR FATHER, OUR FATHER**” with the letter *A* and *O* representing the **Alpha** and **Omega** – the **Beginning** and the **End**. This could make, as yet the speculation goes, the *Sator Magic Square* a safe and hidden way for the early Christians to identify themselves and signal their beliefs to each other without the danger of persecution.

5.5 Stamp S172 (6/2): Franklin – Bent Diagonals

Benjamin Franklin was born in Boston, Massachusetts, 17th January, 1706 and was one of the most influential “**Founding Fathers**” of the United States, earning the title of “**The First American**” for his fight for independence. He was also a man of many interests and talents.

In early days he worked as a printer, to become, with the course of the years, a **polymath**, **author**, **politician**, **scientist**, **inventor**, **musician**, **social activist**, **postmaster general**, **statesman** and **diplomat**.

As a *Postmaster*, he was named in 1775 the first United States **Postmaster General**, establishing a postal system that was the basis for the present United States Post Office.

As an *Author* he started to publish the famous **Poor Richard's Almanack** that became at the time very popular reading. Some of the adages there published remain commons citations even at present.

As an *Inventor* and *Scientist*, among many of his inventions, there are: the **Bifocal Glasses**, the **Lighting Rod**, the **Flexible Urinary Catheter**, and the **Glass Harmonica** etc. He also published several studies about Demography, Atlantic Ocean Currents, Electricity, Meteorology, Cooling Concept, etc.

Being a man with a strong character and clear ethical values, he established for himself, yet very young, as a guide, the following **13 virtues** that he continued to follow during his life: **Temperance**, **Silence**, **Order**, **Resolution**, **Frugality**, **Industry**, **Sincerity**, **Justice**, **Moderation**, **Cleanliness**, **Tranquility**, **Chastity** and **Humility**.

In addition to the numerous achievements, Benjamin Franklin also left his name associated to the “*Magic Squares*”. The “*Magic Squares*” of Benjamin Franklin represented in the stamps shows the same sum for the *Rows* and *Columns* but not for the *Main Diagonals*, i.e, it is only a *Semi Magic Square*. However, it possesses other magic properties as those associated with *Bent Diagonals* either *Continuous* or *Wrap-Around* with sum 260.

In the stamp, several **Bent-Up-Rows Diagonals** can be seen in different colours, including **Bent Wrap-Around Diagonals**.

5.6 Stamp S172 (6/3): Dürer – Melencolia I

Albrecht Dürer, son of a goldsmith, was born in 1471, in Nuremberg, Germany. He became famous as painter, engraver, printmaker, mathematician and academic. He started as an apprentice to Michael Wolgemut when he was young. Later he had been in contact with famous artists like the Schongauer’s brothers, the goldsmiths Caspar and Paul, the painter Ludwig and the sculptor Nikolaus Gerhaert.

Nuremberg was not far away from Venice and Dürer went to Italy twice to study more advance techniques and new artistic expressions. During all these years he could transmit a strong influence and acquire a solid reputation that made him to be regarded as the greatest artist of Northern Renaissance.

After returning to Nuremberg for the second time, he created some famous artistic works as: the paintings, **Adam and Eve** (1507), **The Martyrdom of the Ten Thousand** (1508), **The Virgin with the Iris** (1508), the woodcuts, such as **The First Apocalypses Series** (1498), **The Great Passion** and **The Life of the Virgin** (1511), **The Second Apocalypses Series** (1511), and the well known “Master Prints” (Meisterstiche) **The Knight**, **The Death and The Devil** (1513), **Saint Jerome In His Study** and **Melencolia I** (1514).

Melencolia I is an engraving that includes in the upper right corner, under the bell, a *Normal Associated Magic Square* of *Doubly-Order*.

- The two middle *Cells* of the bottom *Row* show the date of the engraving, 1514.
- The *Magic Sum* is $S = 4(4^2+1)/2 = 34$. In addition to the *Rows*, *Columns* and *Main Diagonals*, the sum S is also possible to be obtained in different ways as follows:
  - The four 2×2 Quadrants, e.g., 16+3+5+10 = 34;
  - The Central Square, e.g., 10+11+6+7 = 34;
  - The Corners of the four 3×3 Squares, e.g., 16+2+9+7 = 34;
  - The Corners of the centered 4×2 and 2×4 Rectangles, e.g., 3+2+15+14 = 34 and 5+8+9+12 = 34;
  - The Corners of the two diagonal 2×3 Rectangles, e.g., 2+8+15+9 = 34 and 5+3+12+14 = 34;
  - The two Skewed Squares, e.g., 8+14+9+3 = 34 and 2+12+15+5 = 34;
  - The Latin Cross Shapes, e.g., 3+5+15+11 = 34 and 2+10+14+8 = 34;
  - The Upside-down Cruciform (St. Peter’s Cross) Shapes 3+9+15+7 = 34 and 2+6+14+12 = 34;
- Any Pair of Cells that are symmetric around the Center sum 17.

5.7 Stamp S172 (6/4): Su Hui – Xuan Ji Tu – Palindrome

Su Hui (351 A.D.- ?) was a Chinese poetess that lived in **Former Qin** of the **Sixteen Kingdoms** period. She married **Dou Tao**, a government official who later was sent to defend the northern borders. Far away from her husband, she found out that he had taken a concubine. To console her unhappiness and try to bring him back she composed her **Palindrome Poem**, **Xuan Ji Tu**, an array of 29 *Lines* × 29 *Columns*, with **841 characters**, that can be read at least in **2848 different ways**, namely, forward, backward, horizontally, vertically and diagonally. After reading the poem, *Dou Tao* left his concubine and return to *Su Hui*, and the love between them became very strong and forever.

This stamp is a square only with 15 *Lines* × 15 *Columns* extracted from the central part of the 29 *Lines* × 29 *Columns* square that constitutes the full poem *Xuan Ji Tu.*

Once the poem can be read in so many different ways, for easier understanding on how it can be read, it is necessary to identify some sets of characters, namely:

- The **Internal Red Frame**, i.e., the **Central Red Square** (3×3), without the character 心 (**xin**). It is said that this character did not appear in the original poem and was added later by another scholar;
- The **Black Frame** evolving around the *Central Red Square*;
- The **4 Black Squares** (4×4) at the inner corners of the *Peripheral Red Frame*;
- The **4 Blue Rectangles** (5×4) between the *Black Squares;*
- The **Peripheral Red Frame**;
- The **Diagonals**.

Si Ku Quan Shu (**The Imperial Collection of Four**) and **Shi Yuan Zhen Pin:Xuan Ji Tu** by **Li Wei** are references used for explaining on how to read the poem in different ways.

1. The **Internal Red Frame**, 8 characters.
   Starting from the middle character 詩(**shi**) and read it in anti-clockwise with 4 characters for each sentence, we will get two sentences "詩圖璣璇，始平蘇氏。(shi tu ji xuan，shi ping su shi。)**"**. Starting from the character 蘇(**su**) to read in the same way, we will get another two sentences "蘇氏詩圖，璣璇始平。" (su shi shi tu , ji xuan shi ping).

   Finally, we can get "詩圖璣璇，始平蘇氏。蘇氏詩圖，璣璇始平。" with the meaning of "*Xuan Ji Tu* is composed by *Su Hui* who lived in Shi Ping County(始平縣) and it is the origin of *Palindrome Poem*."

2. The **Black Frame**, 16 characters.
   It includes 16 characters in black colour. Starting from the right lower corner 怨義(**yuan yi**) and read in clockwise with 4 characters for each sentence, we get"怨義興理，辭麗作比，端無終始，詩情明顯。 (yuan yi xing li，ci li zuo bi，duan wu zhong shi，shi qing ming xian。)"with the meaning of "I use beautiful words and phrases weaved in this brocade to express my ethical

complaints and the rationales of them. However, my love for you continues where you can understand it from the deep emotion meanings embedded in this poem”.

Starting from the left upper corner 端無(**duan wu**) and read in clockwise with 5 characters for each sentence, in which the fifth character of last sentence will be repeated as the first character of next sentence. Hence we can get“端無終始詩，詩情明顯怨。怨義興理辭，辭麗作比端。(duan wu zhong shi shi，shi qing ming xian yuan。yuan yi xing li ci，ci li zuo bi duan。)”with the meaning of “I use this poem to express my love to you. The poem contains my obvious discontentment as well. I need to present in the text the rationales behind my ethical complaints. It is because of my love to you that I am using beautiful words and phrases to write this poem.”

Different poems can be extracted starting from different corners and read in 4-characters, 5-characters, clockwise or anti-clockwise etc. It is said that at least 24 poems can be read.

3. The **4 Black Squares** (4×4).
   Each square contains 16 characters. Starting from the right upper corner 思感(**si gan**) and read it in zigzag way, 4 characters for each sentence. We can get“思感自寧，孜孜傷情。時在君側，夢想勞形。(si gan zi ning，zi zi shang qing。shi zai jun ce，meng xiang lao xing。”with the meaning of “Thinking of you and the time we passed make me sad and restless. I cannot sleep because I am missing you, and this makes me thin and pallid”.

   The characters of the *Black Squares* of characters can be read in *Row* by *Row*, *Column* by *Column*, zigzag in clockwise or anti-clockwise. It is said that at least 176, 4-character poems can be read.

4. The **4 Blue Rectangles** (5×4) between the *Black Squares.*
   Each square contains 20 characters. Take the right rectangle as example and starting from the upper right corner 寒歲(**han sui**), read in zigzag way with 5 characters for each sentence, we can get “寒歲識凋松，始終知物貞。顏喪改華容，士行別賢仁。(han sui shi diao song，shi zhong zhi wu zhen。 yan sang gai hua rong，shi xing bie xian ren。)”It means “Pine trees stand firmly in the cold winter. Since you left me, my face grows aging; however, my love for you is eternal just like the pine trees.”

   The characters of the *Blue Rectangles* can be read in *Row* by *Row*, *Column* by *Column*, zigzag, in clockwise or anti-clockwise. It is said that at least 176, 5-character poems can be read.

5. The **Peripheral Red Frame**, 56 characters.

A frame of 56 characters with 8 special **Rhyme Characters**“欽、林、麟、身、潨、沈、神、殷 (qin、 lin、lin、shen、shen、chen、shen、yin)” arranged in corners and mid-point of each side.

For example, starting from upper right corner 欽(**qin**) and read it clockwise till the lower left corner 沙(**sha**) with 7 characters for each sentence, we can get“欽岑幽巖峻嵯峨，潨淵重涯經網羅。林陽潛曜翳英華，沈浮異逝頹流沙。(qin cen you yan jun cuo e，shen yuan chong ya jing wang luo。lin yang qian yao yi ying hua，cheng fu yi shi tui liu sha。)”with the meaning of “The long curve bank of river, the danger ridge of high mountain, the deep of dark pond, make me fear. I feel depress because my letter to you is lost, just like the warm sunlight for beautiful flowers is blocked by dense forests”.

Starting from different *Rhyme Characters* and read in different way, it is said that at least 96 7-character poems can be read.

6. The **Diagonals**, 29 characters each.
   There are two *Main Diagonals* in the brocade. Starting from the 嗟(**jie**) near the upper right corner, and read it diagonally to the lower left corner with 7 characters for each sentences, we can get“嗟中君容曜多欽，思傷君夢詩璇心。氏辭懷感戚知麟，神輕粲散哀春親。(jie zhong jun rong yao duo qin， si shang jun meng shi xuan xin。shi ci huai gan qi zhi lin，shen qing can san ai chun qin。)”With the meaning of “Thinking of you make me pale in face, I can only express my love in my poem and meet you in my dream. Although Spring comes, I am still in low spirit and feel sad.” Starting from different corner and read in different way, it is said that at least 96 7-characters can be read.

Su Hui used a lot of *Rhyme Characters* which are ingenious arranged in the *Xuan Ji Tu*, and because of this arrangement, even when we start from different character and read in different ways, we still can extract a meaningful poem.

5.8 Stamp S172 (6/5): Lee Sallows – Panmagic 3×3

Born in England in 1944, as a boy **Lee Sallows** became interested in short wave radio, after which he was to find work as a technician in various branches of the electronics industry. In 1970 he moved to Nijmegen in the Netherlands where he was employed by the Radboud University as an electronics engineer, until his retirement in 2009.

After developing an interest in recreational mathematics, he became an expert on the theory of *Magic Squares*, a topic to which he contributed several new variations, most notably *Alphamagic* and

Geomagic Squares. **Sallows has an Erdős number of 2**.

Having become strangely attracted to a **formula due to Édouard Lucas** that characterizes the structure of every *3×3 Magic Square* (among them the *Luo Shu*), Sallows speculated that it might contain hidden potential.

This speculation was confirmed in 1977 when he published a paper that correlated every *Magic Square* of *Order 3* with a unique parallelogram on the complex plane. In an improbable move, he then tried substituting the variables in the Lucas formula with geometrical forms, an eccentric notion that led immediately to the invention of *Geomagic Squares*. It turned out to be an unexpected consequence of this find that **Traditional Magic Squares** using numbers **were now revealed as One-dimensional Geomagic Squares**. Other researchers have since then taken notice, among them Peter Cameron who has suggested that "an even deeper structure may lie hidden beyond *Geomagic Squares*".

The stamp is a *Pandiagonal* or *Nasik 2-D Magic Square* of Order 3, or one in which, in addition to *Rows* and *Columns*, all six *Diagonals* are *Magic*, including the 4 so-called *Broken Diagonals*. In this case the *Magic Target* can also be formed by any three of the four corner *Pieces*. This square is of interest because a *Numerical* equivalent is impossible to construct. The possibility of finding a *Geometrical 3×3 Nasik Square* was thus anything but certain, and their initial discovery an event to celebrate. The resort to disjoint *Pieces* (all of them **Pentominoes**) is an indication of the difficulty encountered in finding it.

5.9 Stamp S172 (6/6): La Loubère or Siamese Construction Method

There are several general methodologies to construct *Magic Squares* depending on the class (*Simple*, *Associated*, *Pandiagonal*, etc.) and *Order*. However these general methodologies not always apply for all the *Orders* of a certain class, as it is the case for the smallest Orders (3 and 4) because they are special cases. Through the times several methods for constructing Magic Squares have been created namely the following: **La Loubère** or **Siamese – Bachet de Méziriac – Philippe de la Hire – John Lee Fults – Ralph Strachey – Stairstep – Diagonal – Knight's Move – Lozenge** (John Conway) **– Dürer**, etc.

La Loubère methodology was created by Simon de la Loubère (1693), a French mathematician that learned it as ambassador to Siam, reason why it is also known by *Siamese*.

La Loubère method is one of the most popular to create *Magic Squares* of *Odd Order*. The main characteristic of this method consists in filling the *Cells* of the *Diagonals* in sequential order and moving upward and to the right.

Let's see how it works:

1. First, the middle *Cell* of the *Row*, is filled with number 1;
2. Whenever you reach the top side of the *Square*, move to the bottom *Cell* of the right *Column* and continue to fill the *Diagonal* upward to the right with the numbers in sequential order;
3. Whenever you reach the right side of the *Square*, move to the most left *Cell* of the upper *Row* and continue to fill the *Diagonal* as before;
4. Whenever you reach a *Cell* that is already filled move down one *Cell* and continue to fill the *Diagonal* as before;
5. If you reach to upper right corner *Cell* also move down one *Cell* and continue as 3.

In this stamp the lines over the *Cells* show the numeric sequence for filling the *Cells* according the methodology mentioned at paragraphs 1 to 5.

6. Magic Squares II Issue: Souvenir Sheet, Sheetlet, First Day Cover and Stamps

6.1 Souvenir Sheet B166 (1/1): Method of Knight's Tour

There are several general methodologies to construct *Magic Squares* depending on the **Class** and **Order**.Among them, the following can be mentioned: **La Loubere** or **Siamese**, **Bachet de Meziriac**, **Philippe de la Hire**, **John Lee Fults**, **Ralph Strachey**, **Knight's Tour**, **Dürer**, etc.

In the *Souvenir Sheet* of *Magic Squares II* issue, *Method of Knight's Tour* is used to construct a **Magic Square of Order 16** with a **Closed Circuit** or **Reentrant**.

This method consists, starting in an **Initial Cell**, to which the number 1 is attributed, to fill numeric and sequentially the **Cells**, from 1 to n^2, of a **Square of Order n**, using the characteristic movements of a *Knight Jump*, as in the Chess game.

Once the **Tour** is established, between the **Initial Starting Cell** and the **Final Arriving Cell**, and if be possible to proceed it, i.e., to "jump" from the *Final Arriving Cell* to the *Initial Starting Cell* with a legal Knight movement, the *Tour* is said **Closed** or **Reentrant** and, in this case, the *Initial Starting Cell* can be anyone. On the contrary, the *Tour* is said **Open** or **Non-reentrant**.

When the *Knight Jump* establish a *Tour* that generates a true *Magic Square*, i.e., when the *Rows*, *Columns*, and *Main Diagonals* add up the same *Magic Sum*, it is said that the *Tour* is a **Magic Knight Tour** (**MKT**).

When the *Main Diagonals Sum* is different from the *Rows* and *Columns Sum*, the *Tour* is said a **Semi Magic Knight Tour** (**SMKT**).

The history to try, by the *Knight Tour Method* to reach all the *Cells* of the **Chess Board**, *8 × 8*, or *Boards* with different Dimensions, *n × n* or *m × n*, in only one *Tour*, comes from Antiquity, while the tentative to create Magic Squares is very much recent, although it is possible to mention several tentatives, among them, by famous mathematicians like **Abraham De Moivre** (1667-1754), **Leonhard Euler** (1707-1783) and **Adrien-Marie Legendre** (1752-1833).

To *De Moivre* is attributed the prowess to be the first to establish a *Tour*, in 1722, although *Open*, able to touch all *Chess Board Cells*, *8 × 8*. Years later, the same endeavor, but in a more complex way, since the *Tour* is *Closed*, could be achieved by *Euler* and *Legendre*, although not *Magic*. *Euler* was among the first to study *Knight Tours* systematically in a scientific way, around 1759. He also created one of the first methods for finding them. However, the best-known historical procedure (**Warnsdorff's Rule**) was created by H.C. Warnsdorff, a German mathematician, in 1823.

Regarding *Semi Magic Squares Knight Tours*, others, as: **William Beverly**, who was the first to publish, in 1847, a *Semi Magic Square* of *Order 8* with an *Open Tour*; **Carl Wenzelides** who published, in 1849, a *Semi Magic Square* of *Order 8* with a *Closed Tour*; **Krishnaraj Wadiar**, who published, in 1852, a *Semi Magic Square* of *Order 8* with a *Closed Tour*; **Carl F. Jaenisch** who published, in 1859, a *Semi Magic Square* of *Order 8* with a *Closed Tour*; **M. A. Feisthamel** who published, in 1884, a *Magic Square* of *Order 8* with an *Open Tour*; are distinguished in the efforts to create, for the *Order 8* or, the same to say, for the *Chess Board*, a *Magic Square* with a *Closed Tour*. This objective comes to be proved impossible to fulfill in August 2003 (**Guentar Stertenbrink** 2003) through the complete computational enumeration of all possibilities. However, during the process 140 different *Semi Magic Tours* were discovered.

The interest aroused by the creation of *Magic Squares* using the *Knight Tour Method* in different dimension *Boards*, led to studies that concluded not to be possible to exist a *Magic Square Tour* in *Boards n × n*, with *n Odd*, although it is feasible for *Boards* of **Order 4 k × 4 k**, with **k>2**.

Among these are to be mentioned the first 4 **Magic Squares of Order 12**, although with an *Open Tour*, created by **Awani Kumar** in 2003 and published in the **Games and Puzzles Journal Issue 26**. Without answer yet remains the question about the existence or not of *Magic Squares of Order 12* with a *Closed Tour*.

Finally, is introduced the **Magic Square of Order 16**, with a **Closed Tour**, the Knight can jump from Cell 256 to 1, that constitutes the design of the **Souvenir Sheet**, published by the author **Joseph**

S. Madachy, in 1979, in the book **Madachy's Mathematical Recreations**.

As can be verified in the *Souvenir Sheet* the attribution of the numbers 1 to 256 in the *Cells* is made sequentially and respects the *Knight Jump* rule on the game of Chess. The *Magic Sum* is 2056.

6.2 Sheetlet

As occurred in *Magic Squares* I, the **Sheetlet** presents a disposition for the face values of the stamps (1 to 9 "patacas") equal to the disposition that the numbers 1 to 9 occupy in the *Luo Shu Magic Square*.

In this issue, the last three stamps, with the **face values** of **8**, **1** and **6** "**patacas**", are issued, corresponding to the **Inferior Row**, as mentioned in the previous *Introductory Note*.

On the **Lateral Margins** two *Magic Squares* **Tiling Schemes** proposed by **David Harper** are shown, which are based in the correspondent between the **decimal** and **binary numerical bases**.

Considering that it is a Configuration with 16 motifs (1 to 16) it is suitable to **Tile** *Order 4 Magic Squares* and their continuous repetitions.

On the **Right Margin**, the *Scheme* takes as the basis a **Right Triangle**, while on the **Left Margin** a **Square**, both with an area equal to 1/4 of the *Cell* area.

When, in an *Order 4 Magic Square* and their extensions, the numbers 1 to 16 are substituted by the equivalent **Tiles**, **Patterns** of great beauty can be obtained.

6.3 First Day Cover ENA195/ENB168: Yang Hui Magic Circles

Not too much is known about ancient **Chinese Mathematicians**, because under the instructions of **Qin Shi Huang** (秦始皇) (259-210 BC), not only some books were burnt but also many scholars were killed (213 BC).

Among the Mathematical Classic works, **Jiu Zhang Suan Shu** (九章算術)**, Nine Chapters of the Mathematical Art** (10 BC-2) is probably the greatest. It is composed, as the title suggests, by **9 Chapters** with **246 problems** covering practical life aspects as: **Weights**, **Measures**, **Surveying**, **Tax Collection**, etc. and **Linear Equations**.

It was only during the Tang Dynasty (618-907) that the most important mathematical works, until then known, were compiled (656), latter known as **Suan Jing Shi Shu** (算經十書), the **Ten**

Computational Canons.

The XIII Century was probably one of the most relevant periods in the History of Chinese Mathematics, with the publication of **Shu Shu Jiu Zhang** (數書九章**), Mathematical Treatise in Nine Sections**, 1247, by **Qin Jiu Shao** (秦九韶) and **Ce Yuan Hai Jing** (測圓海鏡), **Sea Mirror of Circle Measurements**, by **Li Ye** (李冶), followed 15 years later, by the works of **Yang Hui** (楊輝).

Yang Hui (*楊輝*), (1238-1298), born in Qiantang (錢塘) (modern Hangzhou (杭州)), Zhejiang Province (浙江省), during the late Song Dynasty (宋朝) (960-1279), learned mathematics from the works of **Liu Yi** (劉益) who was a native of Zhongshan in Hebei Province (河北中山府).

Among his works, it is relevant to mention the followings:

- 1261, **Xiangjie Jiu Zhang Suanfa** (詳解九章算法) **– Detailed Analysis of the Mathematical Rules in the Nine Chapters and their Reclassifications**;
- 1262, **Riyong Suanfa** (日用算法) **– Mathematical Methods for Daily Use**;
- 1274, **Chengchu Tong Bian Suanbao** (乘除通變算寶) **– Alfa and Omega of Variations and Multiplication and Division,** which consisted of the following 3 volumes: Suanfa Tongbian Benmo Juan Shang (算法通變本末卷上), Vol. I**, Fundamental Changes in Calculation**；Chengchu Tongbian Suanbao Juan Zhong (乘除通變算寶卷中), Vol. II**, Computational Treasure for Variations in Multiplications and Divisions**；and Fasuan Quyong Benmo Juan Xia (法算取用本末卷下), **Vol. III, Fundamentals of the Applications of Mathematics**.
- 1275, **Tianmu Bilei Chengchu Jiefa** (田畝比類乘除捷法) **– Practical Mathematical Rules for Surveying**, Vol. I and Vol. II.
- 1275, **Xugu Zhaiqi Suanfa** (續古摘奇算法) **– Continuation of Ancient Mathematical Methods for Elucidating Strange Properties of Numbers,** Vol. I and Vol. II.

Subsequently, these last three series of *Yang Hui's* works, consisting of 7 volumes, were later assembled and published (1378) in what is now known by **Yang Hui Suanfa** (楊輝算法), **Yang Hui's Methods of Computation**.

The topics covered by *Yang Hui* include Multiplication, Division, Root-extraction, Quadratic and System Equations, Series, Computations of areas of Polygons as well as Magic Squares, Magic Circles, the Binomial Theorem and, the best known work, his contribution to the **Yang Hui's Triangle**, (discovered by his predecessor **Jia Xian** (賈憲)), four hundred years later rediscover by the French Mathematician **Blaise Pascal** (1653).

At the Bottom Left Corner of the *First Day Cover* is presented the *Yang Hui Magic Circles*.

These **Nine Circles** are composed by **72 Numbers**, from 1 to 72, having each individual Circle **8 Numbers**. The neighbouring Numbers make **Four Additional Circles,** also with 8 Numbers each, thus making altogether **13 Circles** in the Square [NW, N, NE, (NW, N, C,W), (N, NE, E, C), W, C, E, (W, C, S, SW), (C, E, SE, S), SW, S, SE], with 8 numbers each, and the following proprieties:

- Total *Sum* of the 72 Numbers = 2628;
- *Sum* of the 8 Numbers in each Circle = 292;
- *Sum* of the 3 Circles along the Horizontal Lines = 876;
- *Sum* of the 3 Circles along the Vertical Lines = 876;
- *Sum* of the 3 Circles along the Main Diagonals = 876.

6.4 Stamp S193 (3/1): McClintock / Ollerenshaw – Most Perfect

It is not possible to establish the history of *Most-Perfect Magic Squares* without to mention **Kathleen Timpson Ollerenshaw**. Despite to be almost completely deaf from early age she could earn a DPhil in Mathematics from Oxford University. Although much of her adult life was devoted to voluntary social services, public education and politics - she was made **Dame Commander of the Order of British Empire** (**DBE**) - she also could take some time to work on Mathematics.

Not only she published a paper in 1980 where she explained one of the first general methods for solving the **Rubik Cube Puzzle**, but also, in 1982, with **Hermann Bondi**, they developed a **mathematical analytical construction** that could verify the number **880** for the **essentially different** *Magic Squares of Order 4* proposed by **Bernard Frénicle de Bessey** in XVII Century.

After this achievement she began to study *Pandiagonal Magic Squares* based on works published by **Emory McClintock** in 1897. After several years, in 1986, *Kathleen Ollerenshaw* published a paper where, making use of *Symmetries,* she proved that there are **368640** *essential different Most-Perfect Magic Squares of Order 8*.

Step by step, she could discover how to construct and how to count the total number of *Most-Perfect Magic Squares* of **Order Power 2**, then for *Magic Squares* whose *Order* is a **Multiple of Power 2**, and finally, for all with an **Order Multiple of 4**.

Together with **David Brée**, who helps her to organize her research notes and proof-reading, they finally, published in 1998 the book "**Most-Perfect Pandiagonal Magic Squares: Their Construction and Enumeration"**. The book received international recognition and has constituted a remarkable accomplishment for a woman of age 85.

Later she said: “I hope to encourage others. The delight of discovery is not a privilege reserved solely for the young.”

As it was previously defined, a *Most-Perfect Magic Square* is a *Pandiagonal Magic Square* of *Doubly-Even Order*, with the additional two proprieties:

- The *Cells* of any square of Order 2, (2 × 2 *Cells*) extracted from it, including *Wrap-Around* sum the same constant value, $2(1 + n^2)$;
- Along the *Main* or *Broken Diagonals*, any two numbers separated by **n/2** *Cells*, are a **Complementary Pair**, i.e. *Sum* $\mathbf{1+n^2}$.

In the case of the *Most-Perfect Magic Square of Order 8* reproduced in the stamp the mentioned properties show the following results:

- $2(1 + n^2) = 2(1 + 8^2) = 130$
  examples: (59 + 38 + 7 + 26) = (48 + 33 + 18 + 31) = (41 + 4 + 32 + 53) include *Wrap-Around* = 130

- $(1 + n^2) = (1 + 8^2) = 65$
  examples: (1 + 64) = (34 + 31) = (25 + 40) = (35 + 30) *Broken Diagonal* = 65

6.5 Stamp S193 (3/2): David Collison – Patchwork

David M. Collison (1937-1991) was born in United Kingdom and lived in Anaheim, California. He was a fruitful creator of *Magic Squares* and **Cubes** to whom, not only is attributed the creation of the **Patchwork Magic Square** presented in this stamp, but also the creation of a **Bimagic Cube of Order 25**, published later by **John R. Hendricks**. He specialized in **Generalized Shapes** from which he created the *Patchwork Magic Squares*.

A *Patchwork Magic Square* is an *Inlaid Magic Square* – one *Magic Square* that contains within it others *Magic Squares*, often placed in the **Quadrants** – that contain *Magic Squares* or **Odd Magic Shapes** within it. The most common *Shape* is **Magic Rectangle**, but **Diamond**, **Cross**, **Elbow** and **L Shapes** can also be found.

These *Shapes* are *Magic* if the *Sum* in each **Direction** is proportional to the number of *Cells*. For example, if a 6 × 8 *Rectangle* has a *Sum* of 120 in the *Short Direction*, the *Sum* in *Long Direction* should be 160. *Main Diagonals* are not required to be the *Magic Sum* unless they belong to a *Square*.

The **Patchwork Magic Square of Order 14** reproduced in this stamp has the following proprieties:

- Contain: Four **Order 4 Magic Squares**, 4 × 4, in the *Quadrants*；one **Magic Cross**, 6 × 6, in the **Centre**；four **Magic Tees**, 6 × 4, on the **Centre Sides**；and four **Magic Elbows**, 4 × 4, in the **Corners**.
- All the *Shapes* sum to a **Constant** directly proportional to the number of *Cells* in a *Row*, *Column* or *Diagonal*: $\mathbf{S_2=197}$；$\mathbf{S_4=394}$；$\mathbf{S_6=591}$；$\mathbf{S_{14}=1379}$. The **Mean** for each *Cell* is **98.5**.

Harvey Heinz, transformed the Four *Magic Squares of Order 4* mentioned above, from **Associated** to **Pandiagonal**, by moving two *Bottom Rows* to the *Top* and then two *Left Columns* to the *Right*.

6.6 Stamp S193 (3/3): Inder Taneja – IXOHOXI 88

Inder Taneja, Professor at the Department of Mathematics, University of Santa Catarina, Brazil, 1978-2012. M.Sc. (1972) and Ph.D. (1975) degrees in Mathematics from Delhi University, India.

Post-doctoral research in Italy (1983) and Spain (1989). Research interests in **Information Theory** especially on Information Measures, Probability of Error, Noiseless Coding, Fuzzy Set Theory, Inequalities, etc.

Recent interest are on **Magic Squares** and **Numbers** Applications and Information Measures to Genetic Code, DNA, etc. Published more than 100 research papers in journals of international reputations. Five chapters in information measures in book series. One online book on Information Measures.

IXOHOXI Magic Squares are a special series that can not only show common properties like other *Magic Squares* and still include alternative properties as **Symmetries**, **Rotations** and **Reflections**.

The word *IXOHOXI* is itself a **Palindrome** and *Symmetric* (*Reflection*), in relation to the “H” centre.

As it can be seen in the stamp, the *Symmetric Properties* not only apply to the Square itself, but also to the numbers of the **7 Segments LED Display** that have been chosen intentionally to fill the *Cells*.

From the 10 digits created with a *7 Segments LED Display*, only 0, 1, 2, 5 and 8, written in digital format, remain the same after a **180 Degree Rotation** (6 becomes 9 and 9 becomes 6). Looking from a **mirror**, these five digits remains the same, with the change that 2 becomes 5 and 5 becomes 2.

It is yet to mention that the 4 digits (**0**, **1**, **2** and **5**) used to construct the **Magic Square of Order 4**, are precisely the same digits that constituted the year **2015**, year of its publication as a stamp.

Taking into consideration the 5 digits and their *Symmetric Properties*, *Inder Taneja* created the *IXOHOXI Universal 88 Magic Square* reproduced in this stamp with the following properties:

1. The *Magic Square* still remains a *Magic Square*:
- After a **Rotation of 180 degree**;
- If it is **seen in a mirror**, or **reflected in water** or **seen from the back** of the sheet;
- The **Magic Sum S** of the *Magic Square of Order 4* is equal to **88**, number that also enjoys *Symmetrical* properties.

2. Additionally working with a *Magic Square of order 5 × 5* using the digits 0, 1, 2, 5 and 8 forms a 176 (88+88). In this case the *Magic Square* "*IXOHOXI Universal 88+88*" is *Pandiagonal*.

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Inder Taneja – Stamp S193 (3/3): Inder Taneja – IXOHOXI 88
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